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SOME REMARKS ON THE EQUALITY $W(E,F^*) = K(E,F^*)$

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Abstract. We show that the equality $W(E,F^*) = K(E,F^*)$ is a necessary condition for the validity of certain results about isomorphic properties in the projective tensor product $E \otimes_\pi F$ of two Banach spaces under some approximation property type assumptions.

Let E, F be two Banach spaces; by $L(E,F), W(E,F), K(E,F)$ we denote the usual Banach spaces of all linear and bounded, weakly compact, compact operators from E to F,respectively.

Several many results concerning lifting of certain isomorphic properties from two Banach spaces E, F to their projective tensor product $E \otimes_\pi F$ contain an assumption of coincidence of the spaces $L(E,F^*), W(E,F^*), K(E,F^*)$ (see [E1], [E2], [E5], [EH], [GG], [R]). Simple examples show that it cannot be dropped at all (see for instance the remark following Theorem 5 below), whereas in some cases (see the papers quoted above) it has been proved that it is really necessary for the validity of those results. Here we want to present some other theorems stating that the equality $L(E,F^*) = W(E,F^*) = K(E,F^*)$ must necessarily hold true if one wants to get a positive lifting result concerning $E \otimes_\pi F$, under quite general assumptions of approximation property type on the spaces involved in the construction. In particular we improve some of the results known.

Before starting our study, we wish to remark that the equality $L(E,F^*) = W(E,F^*)$ is (almost surely) implicitely verified, because of the nature of the properties considered (like property (V) of Pelczynski or the Grothendieck property) without any other assumption on E and F; this means that the *"real"* hypothesis made in the papers quoted above is the equality $W(E,F^*) = K(E,F^*)$. For this reason, we concentrate our attention only on the question of the coincidence of the spaces $W(E,F^*)$ and $K(E,F^*)$. This will allow us to use the weak compactness of the considered operators as a fundamental tool in our proofs to get very general theorems; we do not know if similar results on the equality $L(E,F^*) = K(E,F^*)$ could be obtained in the same generality without this assumption of weak compactness.

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We also observe that again the coincidence of the families of weakly compact and compact operators has been used in [Le] to get the weak sequential completeness of $K(E,F)$ and in [E3] to show that property (V*) of Pelczynski lifts from E^* and F to $K(E,F)$.

In the paper we shall make use of the following notions

Definition 1. We say that a Banach space E has the (b.c.a.p.) if there is a bounded net (T_α) of compact operators from E into itself such that $T_\alpha(e) \to e$,for all $e \in E$, in the norm of E.

Definition 2. We say that a Banach space E has an (u.c.e.i.) if there is a bounded sequence (B_n) of compact operators from E into itself such that $\sum B_n(e) = e$, for all $e \in E$, unconditionally in the norm of E.

Our first results will be consequence of the following general fact

Theorem 1. *Let E, F be two Banach spaces such that $K(E,F)$ is weakly sequentially complete. Suppose F has the (b.c.a.p.). Then $K(E,F) = W(E,F)$.*

Proof. Let $T \in W(E,F)/K(E,F)$ be. This implies that there is a separable subspace E_0 of E such that the restriction T_0 of T to it is not compact; a result in [HM] allows us to suppose that there is an isometric embedding j of E_0^* into E^*. Clearly, $F_0 = \overline{\text{span}}[T_0(E_0)]$ is a separable subspace of F. Since F has the (b.c.a.p.), there is a sequence $(T_n) \subset K(F,F)$ such that

$$T_n(f_0) \to f_0$$

in the norm of F, for all $f_0 \in F_0$. Hence, for all $f^* \in F^*$ and $f_0 \in F_0$, we have that

$$T_n^*(f^*)(f_0) \to f^*(f_0).$$

Hence,for all $e_0 \in E_0$ and $f^* \in F^*$, we get

$$T_n^*(f^*)(T_0(e_0)) \to f^*(T_0(e_0))$$

from which

$$T_0^* T_n^*(f^*)(e_0) \to T_0^*(f^*)(e_0)$$

follows, always for all $e_0 \in E_0$ and $f^* \in F^*$. The last limit relationship means that, for all $f^* \in F^*$,

$$T_0^* T_n^*(f^*) \xrightarrow{w^*} f^*$$

in E_0^*. Recall now that T_0 is a weakly compact operator; this gives that, for all $f^* \in F^*$,

$$T_0^* T_n^*(f^*) \xrightarrow{w} T_0^*(f^*)$$

in E_0^*. Hence, for all $f^* \in F^*$ and $e_0^{**} \in E_0^{**}$ we get

$$T_0^* T_n^*(f^*)(e_0^{**}) \to T_0^*(f^*)(e_0^{**}).$$

Since j^* takes E^{**} into E_0^{**}, for any $f^* \in F^*$ and $e^{**} \in E^{**}$ we get

$$T_0^* T_n^*(f^*)(j^*(e^{**})) \to T_0^*(f^*)(j^*(e^{**}))$$

which gives, for all $f^* \in F^*$ and $e^{**} \in E^{**}$,

$$jT_0^* T_n^*(f^*)(e^{**}) \to jT_0^*(f^*)(e^{**}).$$

We also remark that $jT_0^* T_n^*$ takes F^* into E^* and it is a conjugate operator since $T_0^* T_n^*$ is a weak*-weak continuous operator, because of the weak compactness of T_0; similarly, jT_0^* is a conjugate operator. There are operators $Q_n \in K(E,F), n \in N, Q_0 \in W(E,F)$ such that $Q_n^* = jT_0^* T_n^*, n \in N, Q_0^* = jT_0^*$. Hence,for all $e^{**} \in E^{**}$ and all $f^* \in F^*$ we get

$$Q_n^*(f^*)(e^{**}) \to Q_0^*(f^*)(e^{**})$$

which means ([K]) that (Q_n) is a weak Cauchy sequence in $K(E,F)$. The weak sequential completeness of $K(E,F)$ implies that $Q_0 \in K(E,F)$ and hence that jT_0^* is compact,too. Since j is an isometry, we must necessarily have that T_0 is compact. A contradiction that finishes the proof. □

It is not difficult to see that in Theorem 1 it is enough to suppose there is a weakly compact (but not compact) operator T, that factorizes through a reflexive space with the (b.c.a.p.), instead of assuming that F itself enjoys that approximation property.

For the definitions of the properties involved in the next consequences of Theorem 1 we refer to [D]

Corollary 2. *Let us suppose that $E \otimes_\pi F$ is a Grothendieck space and that F^* has the (b.c.a.p.). Then $L(E,F^*) = W(E,F^*) = K(E,F^*)$.*

Proof. It is enough to observe that if $E \otimes_\pi F$ has the Grothendieck property, then necessarily at least one of the two spaces E, F must be reflexive ([E1],[GG]); this fact implies that $L(E,F^*) = W(E,F^*)$. Furthermore, it is well known that $L(E,F^*) = (E \otimes_\pi F)^*$ is weakly sequentially complete, so that Theorem 1 can be applied. □

Corollary 2 improves a recent result obtained by Gonzalez and Gutierrez in [GG] under the additional assumption *"F is reflexive"*. We remark that from results in [E1] and [GG] it follows,as already quoted, that if $E \otimes_\pi F$ is a Grothendieck space at least one of the two spaces E and F must be reflexive; but it can happen that it is a space without (b.c.a.p.) in the dual, so that our Corollary 2 is more general than that from [GG]. For instance, assume E is a closed subspace Z of some $l_p, 1 < p < 2$, without the (b.c.a.p.) (see [Sz]) and $F = l_\infty$. It is well known that F^* has the (b.c.a.p.). Furthermore, we observe that Z contains a closed subspace H isomorphic to l_p and complemented in l_p and hence also in Z([LT,I]); if we denote by P such a projection, by ϕ the isomorphism between H and l_p, by i the embedding of l_p into l_2 and by j the embedding of l_2 into $F^* = l_\infty^*$, it is easy to show that the operator $ji\phi P$ is a not compact operator from E to F^*; hence, the space $Z \otimes_\pi l_\infty$ is not a Grothendieck space, a fact that does not follow from the result in [GG] since neither Z nor l_∞ may be assumed as F in that theorem.

Corollary 3. *Let us suppose that $E \otimes_\pi F$ has property (V) and that F^* has the (b.c.a.p.). Then $L(E, F^*) = W(E, F^*) = K(E, F^*)$.*

Proof. It is well known that if $E \otimes_\pi F$ has property (V), then $L(E, F^*) = W(E, F^*)$ and $L(E, F^*) = (E \otimes_\pi F)^*$ is weakly sequentially complete, so that again Theorem 1 can be applied. □

Corollary 3 extends a result obtained in [EH] thanks to the existence of (reflexive) Banach spaces possessing the (b.c.a.p.), but not the (b.a.p.) (see the recent [W]).

Corollary 4. *Let E be a reflexive Banach space such that there exists a basic sequence in E^* verifying a upper 2-estimate. Let us suppose that $E \otimes_\pi F$ is a Grothendieck space and that either E^* or F^* has the (b.c.a.p.). Then $E \otimes_\pi F$ is reflexive.*

Proof. Note first that our assumption on E^* is equivalent to the existence of an operator $A : l_2 \to E^*$ taking the unit vector basis of l_2 into a basic sequence. We also observe that F cannot contain copies of l_1, otherwise F^* would contain copies of l_2, a fact that, joined with our assumption on E^* would imply that c_0 lives inside $L(E, F^*)$ (see for instance [E4]); but this would be a contradiction. So F does not contain copies of l_1 and is a Grothendieck space; hence, it must be reflexive ([E1],[GG]). On the other hand, Corollary 2 gives that $L(E, F^*) = W(E, F^*) = K(E, F^*)$, which in turn implies that $L(E, F^*)$ must be reflexive (see for instance [R]). □

Since it is known ([E1],[GG]) that there exist pairs of spaces E, F with E reflexive and F not reflexive for which $E \otimes_\pi F$ is a Grothendieck space, from Corollary 4 it follows that the dual of such a space E cannot contain basic sequences with upper 2-estimates. This remark can be, for instance, applied to the product $T^* \otimes_\pi l_\infty$ (T is the Tsirelson space) that it is known to be a Grothendieck space ([GG]).

In passing we observe that Theorem 1 also improves a necessary condition for $K(E, F)$ to be weakly sequentially complete obtained in [Le] and that it also gives a (partial) converse to a result in [E3] about property (V*) of Pelczynski in $K(E, F)$.

There is another property that lifts from two spaces E, F to $E \otimes_\pi F$ under an assumption of coincidence of all operators from E into F^* with compact operators

Theorem 5. *Let us suppose that E, F do not contain complemented copies of l_1. If $L(E, F^*) = W(E, F^*) = K(E, F^*)$, then $E \otimes_\pi F$ does not contain complemented copies of l_1.*

Proof. If $E \otimes_\pi F$ contains complemented copies of l_1, c_0 must embed into $L(E, F^*) = K(E, F^*)$, which means that $K(E, F^*)$ is uncomplemented into $L(E, F^*)$ (see [E4],[J]).

The assumption $L(E, F^*) = W(E, F^*) = K(E, F^*)$ in the previous Theorem 5 cannot be dropped at all; indeed, if E, F both contain copies of l_1 (not complemented in order to satisfy the other assumptions of Theorem 5), we know that l_2 must live inside both E^* and F^*, a fact implying that c_0 embeds into $K(E, F^*)$(see for instance [E4]). □

The assumption $L(E, F^*) = W(E, F^*) = K(E, F^*)$ in Theorem 5 is sometimes necessary for l_1 to embed complementably in $E \otimes_\pi F$, as we are going to prove in the final part of the paper; our final results will be corollaries of the following

Theorem 6. *Let F be a space with (b.c.a.p.). Suppose also that each separable subspace F_0 of F can be isometrically embedded into a Banach space Z_0 with an (u.c.e.i.) and that $W(E,F) \neq K(E,F)$. Then c_0 embeds into $K(E,F)$.*

Proof. Let us choose a $T \in W(E,F)/K(E,F)$ and a separable subspace E_0 of E such that $T_0 = T_{|E_0}$ is not compact and there is an isometric embedding j of E_0^* into E^* ([HM]). Since F has the (b.c.a.p.) there exists a sequence $(T_n) \subset K(F)$ for which, for all $e_0 \in E_0$,

$$T_n T_0(e_0) \to T_0(e_0)$$

in the norm of F. If we consider a separable subspace F_0 of F containing the ranges of $T_n T_0$ for all $n \in N$ and that of T_0 too, we know that there is an isomorphic embedding h of it in some Z_0 possessing an (u.c.e.i.) (B_n). Clearly, for all $e_0 \in E_0$ we get

$$hT_n T_0(e_0) \to hT_0(e_0)$$

in the norm of Z_0, which means that, for all $z_0^* \in Z_0^*$,

$$T_0^* T_n^* h^*(z_0^*) \xrightarrow{w^*} T_0^* h^*(z_0^*)$$

in the space E_0^*. Since T_0 is weakly compact, for all $z_0^* \in Z_0^*$ we get

$$T_0^* T_n^* h^*(z_0^*) \xrightarrow{w} T_0^* h^*(z_0^*)$$

in the space E_0^* and hence, for all $z_0^* \in Z_0^*$,

$$jT_0^* T_n^* h^*(z_0^*) \xrightarrow{w} jT_0^* h^*(z_0^*)$$

in the space E^*. It follows that

$$jT_0^* T_n^* h^*(z_0^*)(e^{**}) \to jT_0^* h^*(z_0^*)(e^{**})$$

for all $z_0^* \in Z_0^*, e^{**} \in E^{**}$. As in Theorem 1 we can find operators $Q_n \in K(E, Z_0)$, $Q_0 \in W(E, Z_0)$ so that $Q_n^* = jT_0^* T_n^* h^*$, for all $n \in N$, and $Q_0^* = jT_0^* h^*$. Clearly, for all $e \in E$ we have

$$Q_0(e) - \sum_{i=1}^{n} B_i Q_0(e) \to 0$$

unconditionally in the norm of Z_0; this implies ([F]) that $\sum B_i Q_0$ is weakly unconditionally converging; but it is clear that such a series is not unconditionally converging, otherwise Q_0 would be compact. Repeating arguments similar to the previous ones we get

$$Q_0^*(z_0^*)(e^{**}) - \sum_{i=1}^{n} Q_0^* B_i^*(z_0^*)(e^{**}) \to 0$$

for all $z_0^* \in Z_0^*, e^{**} \in E^{**}$. From all of the above limit relationships and the definition of Q_n and Q_0 we easily get ([K]) that

$$Q_n - \sum_{i=1}^{n} B_i Q_0 \xrightarrow{w} 0$$

in $K(E, Z_0)$. It is now possible (see the proof of Proposition 1.c.3 in [LT,II]) to find a series, say $\sum G_n$, of convex combinations of the Q_n's, that is weakly unconditionally converging,but not unconditionally converging in $K(E, Z_0)$. We also observe that $jT_0^*T_n^*$ are conjugate operators; so there are operators $A_n \in K(E, F_0), n \in N$, so that $A_n^* = jT_0^*T_n^*, n \in N$. It is clear that the series got from the A_n^*'s taking convex combinations of them with the same indexes and coefficients used to construct the series $\sum G_n$ is weakly unconditionally converging, but not unconditionally converging in $K(E, F_0)$ because the existence of the embedding h gives that such a space of compact operators actually is isometrically isomorphic to a subspace of $K(E, Z_0)$. Since $K(E, F_0)$ is also a closed subspace of $K(E, F)$, we are done. □

This Theorem 6 answers a question put in [EJ] where a similar result was proved under the assumption: *E is a separably complemented Banach space.*

As a consequence of this result we get a first necessary condition for $E \otimes_\pi F$ do not have complemented copies of l_1.

Corollary 7. *Let us suppose $E \otimes_\pi F$ does not contain complemented copies of l_1. Suppose that at least one of the following conditions is verified:*
1) either E or F is reflexive
2) either E or F is a Banach lattice.
If F^* verifies an assumption like that one considered in Theorem 6 about F, then $L(E, F^*) = W(E, F^*) = K(E, F^*)$.

Proof. If (i) is true, then $L(E, F^*) = W(E, F^*)$. If (ii) is verified, the same equality holds true because of the following reasonings. The remark following Theorem 5, shows that if $E \otimes_\pi F$ does not contain complemented copies of l_1 either E or F must contain no copies of l_1. We can assume E does it (in case F does not contain copies of l_1 we can repeat the following procedure with obvious changes). Suppose, first, that E is a Banach lattice. It is known (see [GJ]) that each operator from E into F^* must be weakly compact, since F^* does not contain copies of c_0. If, instead, we suppose that F is a Banach lattice, then we have that F^* is weakly sequentially complete and so again each operator from E into F^* must be weakly compact; in both cases we have that $L(E, F^*) = W(E, F^*)$. To get the further equality $L(E, F^*) = W(E, F^*) = K(E, F^*)$ it is now enough to apply Theorem 6. □

The same conclusion of Corollary 7 can be obtained under some different assumption on E and F as we shall prove soon; first we state the following result of independent interest in our opinion (we refer to [LT,II] for the definition of property (u))

Theorem 8. *Let E and F be two Banach spaces such that E^* and F are weakly sequentially complete. Suppose also that each separable subspace F_0 of F can be isometrically embedded into a Banach space Z_0 with an (u.c.e.i.). Then $K(E, F)$ has property (u).*

Proof. Let us consider a weak Cauchy sequence $(T_n) \subset K(E, F)$. For all $e \in E$ there is $T(e) \in F$ such that $T_n(e) \xrightarrow{w} T(e)$, thanks to the weak sequential completeness of F. We have so defined a $T \in L(E, F)$. Let us consider a separable subspace F_0 containing the ranges of each T_n;it is clear that also T takes values in

F_0. Denote by h the embedding of F_0 in some Z_0 possessing an (u.c.e.i.) (B_n). For all $e \in E$ we have

$$hT(e)=\sum B_n hT(e)$$

unconditionally in Z_0; hence, for all $e \in E$, we get

$$hT_n(e)-\sum_{i=1}^{n} B_i hT(e) \xrightarrow{w} 0$$

in Z_0. This implies that, for all $z_0^* \in Z_0^*$,

$$T_n^* h^*(z_0^*)-\sum_{i=1}^{n} T^* h^* B_i^*(z_0^*) \xrightarrow{w^*} 0$$

in E^*.Now, we observe that $(T_n^*(f_0^*))$ is weakly Cauchy in E^* and hence weakly converging, for all $f_0^* \in F_0^*$; hence, for all $z_0^* \in Z_0^*$, $(T_n^* h^*(z_0^*))$ is also weakly converging in E^*. Furthermore,$\sum B_n^*(z_0^*)$ is weakly unconditionally converging in Z_0^*, for all $z_0^* \in Z_0^*$; since E^* does not contain c_0, $\sum T^* h^* B_n^*(z_0^*)$ is unconditionally converging in E^*, for all $z_0^* \in Z_0^*$. All together , these facts give that, for all $z_0^* \in Z_0^*$,

$$T_n^* h^*(z_0^*)-\sum_{i=1}^{n} T^* h^* B_i^*(z_0^*) \xrightarrow{w} 0$$

in E^*. So,for all $e^{**} \in E^{**}$ and $z_0^* \in Z_0^*$, we get

$$T_n^* h^*(z_0^*)(e^{**})-\sum_{i=1}^{n} T^* h^* B_i^*(z_0^*)(e^{**}) \to 0.$$

This means that $hT_n-\sum\limits_{i=1}^{n} B_i hT \xrightarrow{w} 0$ in $K(E,Z_0)$ (see [K]). Repeating the proof of Proposition 1.c.3 in [LT,II] we may find a weakly unconditionally converging series built using suitable convex combinations of the T_n's which verifies the condition of property (u) in the space ($K(E,F_0)$ and hence in) $K(E,F)$. We are done. □

In case E and F are also reflexive Banach spaces satisfying the other assumptions of Theorem 8 we get that $K(E,F)$ does not contain copies of l_1 (see [R]) and hence it enjoys property (V) of Pelczynski.

Theorem 8 improves results contained in [Se]. The assumption on subspaces of F we considered in Theorem 8 is sometimes necessary for $K(E,F)$ to possess property (u); indeed, we have the following result which proof can be performed with suitable changes in the proof of the analogous result in [Se]

Theorem 9. *Let us suppose E is a reflexive Banach space with the (b.c.a.p.) such that $K(E)$ has property (u). Then any separable subspace E_0 of E is contained in a Banach space having the (u.c.e.i.).*

Proof. Choose a separable subspace E_0 of E; it is well known that there is a separable subspace E_1 of E containing E_0 and norm-one complemented into E.

Then $K(E_1)$ is a subspace of $K(E)$ and hence it has property (u). Now choose a sequence $(T_n) \subset K(E_1)$ such that $T_n(e_1) \to e_1$ in the norm of E_1. As in Theorem 1, we can prove that it is a weak Cauchy sequence. It follows from the definition of property (u) that there is a weakly unconditionally converging series $\sum B_n$ in $K(E_1)$ such that $T_n - \sum_{i=1}^{n} B_i \xrightarrow{w} 0$, so that $\sum_{i=1}^{n} B_i(e_1) \xrightarrow{w} e_1$. On the other hand, since $e_1 \otimes e_1^* \in K(E_1)^*$, by the definition of weakly unconditionally converging series it follows that $\sum_{i=1}^{+\infty} |B_i(e_1 \otimes e_1^*)| < +\infty$; this fact means that $\sum_{i=1}^{+\infty} B_i(e_1)$ is weakly unconditionally in E_1, a reflexive Banach space; the famous Bessaga-Pelczynski Theorem (see [LT,I],p.98) allows us to conclude that $\sum_{i=1}^{n} B_i(e_1)$ must converge unconditionally, to e_1, of course. $\square$

After this brief digression, we present our last result, which is a further necessary condition for $E \otimes_\pi F$ to possess no complemented copies of l_1; it will be obtained as an application of Theorem 8. Before presenting it we wish to make a remark on Theorem 8. Examples constructed in [Lu] show that it is not possible to eliminate at all any of the hypotheses of Theorem 8. In particular, Theorem 8 and one of the results in [Lu] prove that neither $l_p \otimes_\epsilon l_p, 1 < p < 2$, nor its dual space can be embedded into any Banach space with an (u.c.e.i.), so that they are spaces with basis, but without an (u.c.e.i.).

Corollary 10. *Let us suppose $E \otimes_\pi F$ does not contain complemented copies of l_1 and that E^* and F^* are weakly sequentially complete. Suppose also that either E^* or F^* has the (b.c.a.p.) and that each separable subspace F_0 of F^* can be isometrically embedded into a Banach space Z_0 with an (u.c.e.i.). Then $L(E, F^*) = W(E, F^*) = K(E, F^*)$.*

Proof. We have already remarked that either E or F is not allowed to contain copies of l_1; hence $L(E, F^*) = W(E, F^*)$. Theorem 8 shows that $K(E, F^*)$ has property (u); since it does not contain c_0, it actually is weakly sequentially complete. Now, we may apply Theorem 1 to conclude the proof. $\square$

At the end we observe that if a Banach space has the so called Reciprocal Dunford-Pettis property (see [E2]) or it does not contain copies of l_1, it cannot contain complemented copies of l_1; so, our last results can be used to partially reverse results from the papers [E2] and [E5].

References

[D] Diestel, J., *A survey of results related to the Dunford-Pettis property*, Contemporary Math. 2, Amer. Math. Soc. 1980.

[E1] Emmanuele, G., *About certain isomorphic properties of Banach spaces in projective tensor products*, Extract Math. 5 (1990) 23-25.

[E2] Emmanuele, G., *On the Reciprocal Dunford-Pettis property in projective tensor products*, Math. Proc. Cambridge Phil. Soc. 109 (1991) 161-166.

[E3] Emmanuele, G., *On Banach spaces with the property (V*) of Pelczynski, II*, Annali Mat. Pura Appl. 160 (1991) 163-170.

[E4] Emmanuele, G., *A remark on the containment of c_0 in spaces of compact operators*, Math. Proc. Cambridge Phil. Soc. 111 (1992) 331-335.

[E5] Emmanuele, G., *Banach spaces in which Dunford-Pettis sets are relatively compact*, Archiv Math. 58 (1992) 477-485.

[EH] Emmanuele, G., Hensgen, W., *Property (V) of Pelczynski in projective tensor products*, Proc. Royal Irish Acad. 95A,2 (1995) 227-231.

[EJ] Emmanuele, G., John, K., *Uncomplementability of spaces of compact operators in larger spaces of operators*, Czechoslovak Math. J., to appear.

[F] Feder, M., *On subspaces of spaces with an unconditional basis and spaces of operators*, Illinois J. Math. 24 (1980) 196-205.

[GJ] Ghoussoub, N., Johnson, W. B., *Factoring operators through Banach lattices not containing* $C(0,1)$, Math. Z. 194 (1987) 153-171.

[GG] Gonzalez, M., Gutierrez, J., *Polynomial Grothendieck property*, preprint 1994.

[HM] Heinrich, S., Mankiewicz, P., *Applications of ultrapowers to the uniform and Lipschitz classification of Banach spaces*, Studia Math. 73 (1982) 215-251.

[J] John, K., *On the uncomplemented subspace* $K(X,Y)$, Czechoslovak Math. J. 42 (1992) 167-173.

[K] Kalton, N., *Spaces of compact operators*, Math. Annalen 208 (1974) 267-278.

[Le] D.Lewis, *Conditional weak compactness in certain inductive tensor products*, Math. Annalen 201 (1973) 201-209.

[LT,I] Lindenstrauss, J., Tzafriri, L., *Classical Banach spaces, Sequence Spaces*, EMG 92, Springer Verlag 1977.

[LT,II] Lindenstrauss, J., Tzafriri, L., *Classical Banach spaces, Function Spaces*, EMG 97, Springer Verlag 1979.

[Lu] Lust, F., *Produits tensoriels projectifs d'espaces de Banach*, Colloquium Math. 36 (1976) 255-267.

[R] Ruess, W., *Duality and geometry of spaces of operators, in Functional Analysis: Surveys and Recent Results, III*, Math. Studies 90, North Holland 1984.

[Se] Sersouri, A., *Propriété (u) dans les espaces d'opérateurs*, Bull. Acad. Pol. Sci. 36 (1988) 655-659.

[Sz] Szankowski, A., *Subspaces without the approximation property*, Israel J. Math. 30 (1978) 123-129.

[W] Willis, G., *The compact approximation property does not imply the approximation property*, Studia Math. 103 (1992) 99-108.

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