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COMMUTATIVE NONSTATIONARY STOCHASTIC FIELDS

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Abstract. The present paper is devoted to further development of commutative nonstationary field themes; the first studies in this area were performed by K. Kirchev and V. Zolotarev [4, 5].

In this paper a more complicated variant of commutative field with nonstationary rank 2, carrying into more general situation for correlation function is studied. A condition of consistency (see (7) below) for commutative field is placed in the basis of the method proposed in [4, 5] and developed in this paper. The following semigroup structures of correlation theory for disturbances and semigroups are used in this case: $T_t(\varepsilon) = \exp(itA_\varepsilon)$, $A_\varepsilon = A_1 + \varepsilon A_2$, $|\varepsilon| \ll 1$.

1. In this section we will present the main preliminary information [4, 5].

Let us consider a two-dimensional curve $T_t = \exp(it_1A_1 + it_2A_2)$ in Hilbert space H. From now on we will assume that the system of linear bounded operators $\{A_1, A_2\}$ is a commutative one, $[A_1, A_2] = 0$, and there hold true:

$$\begin{aligned} &1)\quad (A_2)_I H \subset (A_1)_I H\,; \\ (1)\qquad &2)\quad (A_1)_I \geq 0\,; \\ &3)\quad (A_1)_I|_{\overline{(A_1)_I H}} \qquad \text{is restrictedly invertible.} \end{aligned}$$

As it is known [7], the system $\{A_1, A_2\}$ can be included in the commutative colligation

$$\Delta = (A_1, A_2, H, \Phi, E, \sigma_1, \sigma_2, \gamma, \widetilde{\gamma})\,. \tag{2}$$

Where: E is Hilbert space; $\Phi : H \to E$; $\sigma_1, \sigma_2, \gamma, \widetilde{\gamma}$ are selfadjoint operators in E and also the next colligation relationships are valid:

$$\begin{aligned} &1)\quad A_k - A_k^* = i\Phi^*\sigma_k\Phi \qquad (k = 1, 2)\,; \\ (3)\qquad &2)\quad \gamma\Phi = \sigma_1\Phi A_2^* - \sigma_2\Phi A_1^*\,; \\ &3)\quad \widetilde{\gamma} = \gamma + i(\sigma_1\Phi\Phi^*\sigma_2 - \sigma_2\Phi\Phi^*\sigma_1) \end{aligned}$$

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From now on we will study only the case of finite-dimensional space E. From the assumptions (1) follows that we can conclude that $\sigma_1 = I_E$; i.e., A_1 is dissipative. This means that the semigroup T_t when $t_2 = 0$ is contractive. It is evident that when ε is small enough then the operator $A_\varepsilon = A_1 + \varepsilon A_2$ is also dissipative and the semigroup $T_t(\varepsilon) = \exp(itA_\varepsilon)$; $(t_1 = t, t_2 = \varepsilon t)$ is contractive. We will study CF and ICF of semigroup of contractions $T_t(\varepsilon)$ as a function of the variables t and ε. Similarly to [2] it is easy to prove that there exists the limit

$$S \cdot \lim_{t\to\infty} T_t^*(\varepsilon)\, T_t(\varepsilon) = K_\varepsilon \tag{4}$$

and also $0 \le K_\varepsilon \le I$.

Proposition 1. *For every x and s from R there holds true*

$$e^{isA_x^*} K_\varepsilon e^{isA_x} = K_\varepsilon\,, \quad \textit{where} \quad A_x = A_1 + xA_2\,. \tag{5}$$

Proof. It is evident that

$$e^{-isA_x^*} K_\varepsilon e^{isA_x} = s \cdot \lim_{t\to\infty} e^{-i(sA_x^* + tA_\varepsilon^*)}\, e^{i(tA_\varepsilon + sA_x)}$$
$$= S \cdot \lim_{t\to\infty} e^{i\frac{s(x-\varepsilon)}{t+s}A_\varepsilon^*}\, e^{-i(t+s)A_\varepsilon^*}\, e^{i(t+s)A_\varepsilon}\, e^{i\frac{s(x-\varepsilon)}{t+s}A_2} = K_\varepsilon$$

because of strong continuity of semigroup $e^{i\delta A_2}$ which tends to I when $\delta \to 0$. $\square$

Corollary 1. *In this way for every small enough $\varepsilon(|\varepsilon| << 1)$ we can assert that $A_1^* K_\varepsilon = K_\varepsilon A_1$, $A_2^* K_2 = K_\varepsilon A_2$. From the existence of the limit K_ε, (5), there follows that for the correlation function $K_\varepsilon(t,s) = \langle T_t(\varepsilon)\Psi, T_s(\varepsilon)\Psi\rangle$ the next formula is valid*

$$K_\varepsilon(t,s) = V_\varepsilon(t-s) + \int_0^\infty W_\varepsilon(t+\tau, s+\tau)\, d\tau \tag{6}$$

where, as usual, ICF *is defined by the formula*

$$W_\varepsilon(t,s) = -(\partial t + \partial s) K_\varepsilon(t,s)$$

and

$$V_\varepsilon(t-s) = \langle K_\varepsilon e^{i(t-s)A_\varepsilon}\Psi, \Psi\rangle\,.$$

Let us now notice the ICF, *$W_\varepsilon(t,s)$, which, obviously, has the form*

$$W_\varepsilon(t,s) = 2\langle (A_\varepsilon)_I T_t(\varepsilon)\Psi, T_s(\varepsilon)\Psi\rangle = \langle \sigma_\varepsilon \Phi\Psi_\varepsilon(t,\cdot), \Phi\Psi_\varepsilon(s,\cdot)\rangle\,,$$

here $\Psi_\varepsilon(t,\cdot) = T_t(\varepsilon)\Psi$, $\sigma_\varepsilon = \sigma_1 + \varepsilon\sigma_2$, $(A_\varepsilon)_I = (2i^{-1})(A_\varepsilon - A_\varepsilon^*)$.

Proposition 2. *The function $f(\varepsilon,t)=\Phi\Psi_\varepsilon(t)$ is a solution of the equation* (7):

$$[\sigma_2 i\sigma_t - it^{-1}\sigma_\varepsilon^{-1}\sigma_\varepsilon - \widetilde{\gamma}]\, f(\varepsilon,t)=0\,. \tag{7}$$

Proof. As far as $\widetilde{\gamma}\Phi=\sigma_\varepsilon\Phi A_2-\sigma_2\Phi A_\varepsilon$, so

$$\begin{aligned}\widetilde{\gamma} f &= \sigma_\varepsilon\Phi A_2\Psi-\sigma_2\Phi A_\varepsilon\Psi=\sigma_\varepsilon\Phi A_2 e^{itA_\varepsilon}\Psi-\sigma_2\Phi A_\varepsilon e^{itA_\varepsilon}\Psi\\ &=\sigma_\varepsilon(it)^{-1}\Phi\sigma_\varepsilon e^{itA_\varepsilon}\Psi-\sigma_2(i^{-1})\Phi\sigma_t e^{itA_\varepsilon}\Psi=\sigma_2 i\sigma_t f-it^{-1}\sigma_\varepsilon\sigma_\varepsilon f\,,\end{aligned}$$

which proves the assertion. □

Therefore, knowing the function $f(t)$ (when $\varepsilon=0$) we can compute also the function $f(\varepsilon,t)$ as a solution of the next Cauchy problem:

$$\begin{cases} i\sigma_\varepsilon f(\varepsilon,t)=t\sigma_\varepsilon[\sigma_2 i\sigma_t-\widetilde{\gamma}]\, f(\varepsilon,t)\\ f(\varepsilon,t)|_{\varepsilon=0}=f(t)\end{cases} \tag{8}$$

2. Let $\dim E=2$ and the operators $\sigma_1,\sigma_2,\widetilde{\gamma}$ are of the form

$$\sigma_1=\begin{pmatrix}1&0\\0&1\end{pmatrix},\quad \sigma_2=\begin{pmatrix}0&m\\ \overline{m}&\sigma\end{pmatrix},\quad \widetilde{\gamma}=\begin{pmatrix}\alpha&\beta\\ \overline{\beta}&\alpha\end{pmatrix} \tag{9}$$

where $\alpha,\beta\in\mathbb{R}$; $m\in\mathbb{C}$.

In the Hilbert space $L^2_{(0,\ell)}(E^2,dx)$, we consider a model operator system

$$\begin{aligned} A_1 f_x &= i\int_x^\ell f_\zeta\, d\zeta\\ A_2 f_x &= f_x\begin{pmatrix}\alpha&\beta\\ \overline{\beta}&\alpha\end{pmatrix}+i\int_x^\ell f_x\begin{pmatrix}0&m\\ \overline{m}&0\end{pmatrix}d\zeta\,,\end{aligned} \tag{10}$$

where $f_x=(f^1(x),f^2(x))\in L^2_{(0,\ell)}(E^2,dx)$.

Further, we consider a contractive semigroup

$$f(t,x)=T_t(\varepsilon)f_x=e^{itA_\varepsilon}f_x \tag{11}$$

with $1-\varepsilon|m|^2>0$, i.e., $\varepsilon\ll|m|^{-1}$. Then $f(t,x)$ is a solution of the Cauchy problem

$$\begin{cases}\frac{d}{dx}f(t,x)=f_x(t,x)i\varepsilon\begin{pmatrix}\alpha&\beta\\ \overline{\beta}&\alpha\end{pmatrix}-\int_x^\ell f(t,\zeta)\,d\zeta\begin{pmatrix}1&\varepsilon m\\ \varepsilon\overline{m}&1\end{pmatrix}\\ f(0,x)=f_x\end{cases} \tag{12}$$

We introduce in the consideration a vector-function $F(t,x)$ such that

$$F(t,x)=f(t,x)\exp\left\{-it\varepsilon\begin{pmatrix}\alpha&\beta\\ \overline{\beta}&\alpha\end{pmatrix}\right\} \tag{13}$$

Then the Cauchy problem (12) for $F(t,x)$ takes the form

$$\begin{cases} F_t(t,x) = -\int_x^\ell F(t,\zeta)\,d\zeta b_t \\ F(0,t) = f(x) \end{cases} \tag{14}$$

where the matrix b_t has the form

$$\begin{aligned} b_t &= e^{it\varepsilon\begin{pmatrix} \alpha & \beta \\ \overline{\beta} & \alpha \end{pmatrix}} \begin{pmatrix} 1 & \varepsilon m \\ \varepsilon\overline{m} & 1 \end{pmatrix} e^{-it\varepsilon\begin{pmatrix} \alpha & \beta \\ \overline{\beta} & \alpha \end{pmatrix}} \\ &= e^{it\varepsilon\begin{pmatrix} \alpha & \beta \\ \overline{\beta} & \alpha \end{pmatrix}} U \begin{pmatrix} 1+\varepsilon m & 0 \\ 0 & 1-\varepsilon m \end{pmatrix} U^* e^{-it\varepsilon\begin{pmatrix} \alpha & \beta \\ \overline{\beta} & \alpha \end{pmatrix}} \end{aligned}$$

and the unitary matrix U has the form

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$

We calculate the expression

$$\begin{aligned} e^{i\varepsilon t\begin{pmatrix} \alpha & \beta \\ \beta & \alpha \end{pmatrix}} U &= e^{i\varepsilon t\begin{pmatrix} \alpha & 0 \\ 0 & \alpha \end{pmatrix}} e^{i\varepsilon t\begin{pmatrix} 0 & \beta \\ \beta & 0 \end{pmatrix}} U \\ &= \frac{e^{it\varepsilon\alpha}}{\sqrt{2}} \begin{pmatrix} \cos t\varepsilon\beta & i\sin t\varepsilon\beta \\ i\sin t\varepsilon\beta & \cos t\varepsilon\beta \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \\ &= \frac{e^{it\varepsilon\alpha}}{\sqrt{2}} \begin{pmatrix} e^{it\varepsilon\beta} & e^{-it\varepsilon\beta} \\ e^{it\varepsilon\beta} & -e^{-it\varepsilon\beta} \end{pmatrix} \end{aligned}$$

Therefore,

$$\begin{aligned} b_t &= \frac{1}{2} \begin{pmatrix} e^{it\varepsilon\beta} & e^{-it\varepsilon\beta} \\ e^{it\varepsilon\beta} & -e^{-it\varepsilon\beta} \end{pmatrix} \begin{pmatrix} 1+\varepsilon\beta & 0 \\ 0 & 1-\varepsilon\beta \end{pmatrix} \begin{pmatrix} e^{-it\varepsilon\beta} & e^{-it\varepsilon} \\ e^{it\varepsilon\beta} & -e^{-it\varepsilon} \end{pmatrix} \\ &= \begin{pmatrix} 1 & \varepsilon m \\ \varepsilon m & 1 \end{pmatrix} \end{aligned}$$

Thus we obtain b_t is independent on t.

Finally, let us determine an explicit form of $F(t,x)$ (13).

For this purpose we represent $F(t,x)$ in the form

$$F(t,x) = \exp\left\{ t\int_x^\ell \cdot\, d\zeta \right\} \cdot \exp\left\{ -t\varepsilon m \int_x^\ell \cdot\, d\zeta \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \right\} f(x) \tag{15}$$

First we calculate

$$\begin{aligned} &\exp\left(-t\int_x^\ell \cdot\, d\zeta \right) f = \sum_0^\infty \frac{(-t)^n}{n!} \left(\int_x^\ell \cdot\, d\zeta \right)^n f(x) \\ &= f(x) - \frac{t}{n!}\int_x^\ell f_\zeta\, d\zeta + \frac{t^2}{2!}\int_x^\ell (\zeta - x) f_\zeta\, d\zeta - \frac{t^3}{3!}\int_x^\ell \frac{(\zeta-x)^2}{2!} f_\zeta\, d\zeta + \cdots = \end{aligned}$$

Assuming then that f' exists and belongs to $L^2_{(0,\ell)}(E^2,dx)$ with $f_\ell=0$, we obtain after integration by parts

$$\exp\left(-t\int_x^\ell \cdot\, d\zeta\right) = -\int_x^\ell f'_\zeta y_0\left(2\sqrt{t(\zeta-x)}\right)\, d\zeta \tag{16}$$

where $J_0(z)=\sum\limits_0^\infty \frac{(-1)^k(\frac{z}{2})^{2k}}{k!k!}$ the Bessel function of zero order.

Now we calculate

$$\begin{aligned}
&\exp\left\{-t\varepsilon m\int_x^\ell \cdot\, d\zeta\begin{pmatrix}0&1\\1&0\end{pmatrix}\right\} f = \sum_0^\infty \frac{(-t\varepsilon m)^n}{n!}\left(\int_x^\ell \cdot\, d\zeta\begin{pmatrix}0&1\\1&0\end{pmatrix}\right)^n f\\
&= f(x)\frac{t^2\varepsilon^2m^2}{2!}\int_x^\ell(\zeta-x)f_\zeta\, d\zeta+\frac{t^4\varepsilon^4m^4}{4!}\int_x^\zeta\frac{(\zeta-x)^3}{3!}f_\zeta d\zeta+\cdots\\
&\quad-\left\{\frac{t\varepsilon m}{1!}\int_x^\ell f_\zeta\, d\zeta+\frac{t^3\varepsilon^3m^3}{3!}\int_x^\ell\frac{(\zeta-x)^2}{2!}f_\zeta\, d\zeta+\cdots\right\}\begin{pmatrix}0&1\\1&0\end{pmatrix}.
\end{aligned}$$

Selecting as earlier $f(x)$ from a dense set in $L^2_{(0,\ell)}(E^2,dx)$ such that $f'(x)$ exists and is in $L^2_{(0,\ell)}(E^2,dx)$ with $f(\ell)=0$, we obtain after integration by parts

$$\begin{aligned}
\exp\left\{-t\varepsilon m\int_x^\ell \cdot\, d\zeta\begin{pmatrix}0&1\\1&0\end{pmatrix}\right\} f &= -\int_x^\ell f'_\zeta\sum_0^\infty\frac{(t\varepsilon m)^{2k}(\zeta-x)^{2k}}{(2k)!(2k)!}\, dx\\
&\quad+\int_x^\ell f_\zeta\sum_0^\infty\frac{(t\varepsilon m)^{2k+1}(\zeta-x)^{2k+1}}{(2k+1)!(2k+1)!}\, d\zeta\begin{pmatrix}0&1\\1&0\end{pmatrix}.
\end{aligned}$$

Further we make use of

$$J_0(2\sqrt{x})+I_0(2\sqrt{x})=2\sum_0^\infty\frac{x^{2k}}{2k!2k!}$$

where $I_0(x)$ is the modified Bessel function of zero order:

$$I_0(x)=J_0(ix)\,.$$

Finally, we have

$$\exp\left\{-t\varepsilon m\int_x^\ell \cdot\, d\zeta\begin{pmatrix}0&1\\1&0\end{pmatrix}\right\} f = -\frac{1}{2}\int_x^\ell f'_\zeta\, d\zeta \tag{17}$$

$$\begin{pmatrix}J_0(2\sqrt{t\varepsilon m(\zeta-x)})+I_0(2\sqrt{t\varepsilon m(\zeta-x)}); & J_0(2\sqrt{t\varepsilon m(\zeta-x)})-I_0(2\sqrt{t\varepsilon m(\zeta-x)})\\ J_0(2\sqrt{t\varepsilon m(\zeta-x)})-I_0(2\sqrt{t\varepsilon m(\zeta-x)}); & J_0(2\sqrt{t\varepsilon m(\zeta-x)})+I_0(2\sqrt{t\varepsilon m(\zeta-x)})\end{pmatrix}$$

A last, using expressions (16) and (17), we determine the form of function $F(t,x)$ (15).

To this end, it is necessary to calculate the following integrals:

$$
\begin{aligned}
I_1 &= \int_x^\ell J_0(2\sqrt{t(\zeta-x)})\frac{d}{d\zeta}\int_\zeta^\ell f'\eta J_0(2\sqrt{t\varepsilon m(\eta-\zeta)})\,d\eta \\
I_2 &= \int_x^\ell J_0(2\sqrt{t(\zeta-x)})\frac{d}{d\zeta}\int_\zeta^\ell f'_\eta I_0(2\sqrt{t\varepsilon m(\eta-\zeta)})\,d\eta
\end{aligned}
\tag{18}
$$

To simplify the first of them (the second is calculated simplify) we make use of integration by parts:

$$
\begin{aligned}
I_1 &= \int_x^\ell f'_\zeta J_0(2\sqrt{t\varepsilon m(\zeta-x)})\,d\zeta \\
&\quad - \int_x^\ell d\zeta J_{-1}(2\sqrt{t(\zeta-x)})\frac{\sqrt{t}}{\sqrt{\zeta-x}}\int_\zeta^\ell f'_\eta J_0(2\sqrt{t\varepsilon m(\eta-\zeta)})\,d\eta \\
&= \int_x^\ell f'_\zeta J_0(2\sqrt{t\varepsilon m(\zeta-x)})\,d\zeta \\
&\quad - \sqrt{t}\int_x^\ell f'_\eta\,d\eta\int_x^\ell J_{-1}(2\sqrt{t(\zeta-x)})J_0(2\sqrt{t\varepsilon m(\eta-\zeta)})\frac{d\zeta}{\sqrt{\zeta-x}}
\end{aligned}
$$

Using the following formula [1]

$$
\int_0^t \sqrt{\tau-1}J_{-1}(\alpha\sqrt{\tau})J_0(\beta\sqrt{t-\tau}) = 2\alpha^{-1}J_0(t\sqrt{\alpha^2+\beta^2})
$$

we obtain

$$
\int_x^\eta J_{-1}(2\sqrt{t(\zeta-x)}J_0(2\sqrt{t\varepsilon m}))\frac{d\zeta}{\sqrt{\zeta-x}} = \frac{1}{\sqrt{t}}J_0\left((\eta-x)2\sqrt{t+t\varepsilon m}\right)\,.
$$

Thus

$$
I_1 = \int_x^\ell f'_\zeta\left\{J_0(2\sqrt{t\varepsilon m(\zeta-x)}) - J_0\left((\zeta-x)2\sqrt{t+t\varepsilon m}\right)\right\}d\zeta\,. \tag{19}
$$

In a similar manner we obtain

$$
I_2 = \int_x^\ell f'_\zeta\left\{I_0(2\sqrt{t\varepsilon m(\zeta-x)}) - J_0\left((\zeta-x)2\sqrt{t-t\varepsilon m}\right)\right\}d\zeta\,.
$$

Taking into account the expressions (16), (17) and (19), we obtain that the components of vector-function $F(t,x) = (F^1(t,x); F^2(t,x))$ (13) are as follows:
(20)

$$F^1(t,x) = \frac{1}{2}\int_x^\ell \Big\{[\partial\zeta f^1(\zeta) + \partial\zeta f^2(\zeta)](J_0(2\sqrt{t(\zeta-x)}) - J_0((\zeta-x)2\sqrt{t+t\varepsilon m})) + [\partial\zeta f^1(\zeta) - \partial\zeta f^2(\zeta)](I_0\,(2\sqrt{t\varepsilon m(\zeta-x)}) - J_0((\zeta-x)2\sqrt{t+t\varepsilon m}))\Big\}\,d\zeta$$

$$F^2(t,x) = \frac{1}{2}\int_x^\ell \Big\{[\partial\zeta f^1(\zeta) + \partial\zeta f^2(\zeta)](J_0(2\sqrt{t(\zeta-x)}) - J_0((\zeta-x)2\sqrt{t+t\varepsilon m})) + [\partial\zeta f^1(\zeta) - \partial\zeta f^2(\zeta)](I_0(2\sqrt{t\varepsilon m(\zeta-x)}) - J_0((\zeta-x)2\sqrt{t-t\varepsilon m}))\Big\}\,d\zeta$$

Finally it remains to take into account (13):

$$f(t,x) = F(t,x)\exp\left\{it\begin{pmatrix}\alpha & \beta\\ \beta & \alpha\end{pmatrix}\right\} = F(t,x)e^{it\alpha}\begin{pmatrix}\cos t\varepsilon\beta & i\sin t\varepsilon\beta\\ i\sin t\varepsilon\beta & \cos t\varepsilon\beta\end{pmatrix}$$

hence

$$\begin{aligned} f^1(t,x) &= e^{it\alpha}[F^1(t,x)\cos t\varepsilon\beta + F^2(t,x)i\sin t\varepsilon\beta]\\ f^2(t,x) &= e^{it\alpha}[F^1(t,x)i\sin t\varepsilon\beta + F^2(t,x)\cos t\varepsilon\beta]\end{aligned} \tag{21}$$

Accounting the asymptotic of Bessel function [1] $J_0(z)$ when $z \to \infty(|\arg z| < \pi)$ we will obtain that $f(t,x) \to 0$ when $t \to \infty$. So $T_t(\varepsilon)$ is asymptotically decaying function and hence $V_\varepsilon(t-s) = 0$.

Theorem 1. *The limit correlation function $V_\varepsilon(t-s)$ for the stochastic field $T_t(\varepsilon) = \exp it(A_1 + \varepsilon A_2)$, when A_1, A_2 have the form (10) ($|\varepsilon| < \frac{1}{m}$) is equal to zero $V_\varepsilon(t-s) = 0$. Therefore infinitesimal correlation function $W_\varepsilon(t,s)$ has the form*

$$W_\varepsilon(t,s) = (1+\varepsilon m)\Phi^1(t,\varepsilon)\overline{\Phi^1(s,\varepsilon)} + (1-\varepsilon m)\Phi^2(t,\varepsilon)\overline{\Phi^2(s,\varepsilon)} \tag{22}$$

where it is obvious that

$$\Phi^1(t,\varepsilon) = \frac{1}{2}\int_0^\ell (f^1(t,x) + f^2(t,x))\,dx$$
$$\Phi^2(t,\varepsilon) = \frac{1}{2}\int_0^\ell (f^1(t,x) - f^2(t,x))\,dx$$

The Cauchy problem (7) for this case and the function $\Phi(t,\varepsilon) = (\Phi^1(t,\varepsilon), \Phi^2(t,\varepsilon))$ has the form

$$\begin{cases} \partial_\varepsilon\Phi(t,\varepsilon) = t\left\{m\begin{pmatrix}1+\varepsilon m & 0\\ 0 & \varepsilon m - 1\end{pmatrix}\partial_t + 2i\begin{pmatrix}(\alpha+\beta)(1+\varepsilon m) & 0\\ 0 & (\alpha-\beta)(1-\varepsilon m)\end{pmatrix}\right\}\Phi(t,\varepsilon)\\ \Phi(t,0) = (\Phi^1(t), \Phi^2(t))\end{cases} \tag{23}$$

where $\Phi(t,0)$ is determined by a dissipative process with a spectrum at zero and

$$\begin{aligned}\Phi^1(t) &= \frac{1}{2}\int_0^{\ell}(f^1(\zeta)+f^2(\zeta))J_0(2\sqrt{t\zeta})\,d\zeta\\ \Phi^2(t) &= \frac{1}{2}\int_0^{\ell}(f^1(\zeta)-f^2(\zeta))J_0(2\sqrt{t\zeta})\,d\zeta\end{aligned}\tag{24}$$

One can write the equation of Cauchy problem (23) in the following form:

$$\begin{aligned}\Phi^1_\varepsilon &= (1+\varepsilon m)(mt\Phi^1_t + 2i(\alpha+\beta)\Phi^1)\\ \Phi^2_\varepsilon &= (\varepsilon m-1)(mt\Phi^2_t + 2i(\beta-\alpha)\Phi^2)\end{aligned}\tag{25}$$

where $\Phi^k_\varepsilon = \partial_\varepsilon \Phi^k$, $\Phi^k_t = \partial_t\Phi^k$, $(k=1,2)$; therefore it is necessary to solve in a general form the equation

$$\partial_\varepsilon\Phi = (\varepsilon m + a)(mt\partial_t\Phi + ib\Phi)\,,$$

where $a, b, m \in \mathbb{R}$. It is easy to see that a general solution of this equation has the form

$$\Phi(t,\varepsilon) = e^{\frac{ib}{4m}((a+\varepsilon m)^2 - 2\ln t)}G(2\ln t + (\varepsilon m + a)^2)$$

where $G(x)$ is an arbitrary differentiable function.

Taking into account the initial condition of the problem (23), it is easy to obtain

$$\begin{aligned}\Phi^1(t,\varepsilon) &= e^{\frac{i(\alpha+\beta)}{2}\varepsilon(\varepsilon m+4)}\Phi^1(e^{\frac{2t+\varepsilon^2m^2+2\varepsilon m}{2}})\\ \Phi^2(t,\varepsilon) &= e^{\frac{i(\alpha-\beta)}{2}\varepsilon(\varepsilon m-4)}\Phi^2(e^{\frac{2t+\varepsilon^2m^2-2\varepsilon m}{2}})\end{aligned}\tag{26}$$

where $\Phi^k(t)$ has the form of (24).

Thus, the following theorem is proved.

Theorem 2. *The correlation function of stochastic field $T_t(\varepsilon)$ for the commutative system of operators A_1, A_2 (10) has the form*

$$K_\varepsilon(t,s) = \int_0^\infty W_\varepsilon(t+\tau, s+\tau)\,d\tau$$

where $W_\varepsilon(t,s)$ has the form (22) and $\Phi^k(t,\varepsilon)$ is represented in the form (26).

References

[1] Erdelyi, A. (ed.), *Higher transcendental functions*, McGraw-Hill, New York 1953.

[2] Kirchev, K. P., *On a certain class of non-stationary random processes*, Teor. Funkts., Funkts. Anal. Prilozh., Kharkov **14** (1971), 150–160 (Russian).

[3] Kirchev, K. P., *Linear representable random processes*, God. Sofij. Univ., Mat. Fak. **66** (1974), 287–306 (Russian).

[4] Kirchev, K. P., Zolotarev, V. A., *Nonstationary curves in Hilbert spaces and their correlation functions I*, Integral Equations Operator Theory **19** (1994), 270–289.

[5] Kirchev, K. P., Zolotarev, V. A., *Nonstationary curves in Hilbert spaces and their correlation functions II*, Integral Equations Operator Theory **19** (1994), 447–457.

[6] Livshits, M. S., Yantsevich, A. A., *Theory of operator colligation in Hilbert space*, Engl. transl. J. Wiley, N.Y. 1979.

[7] Zolotarev, V. A., *On open systems and characteristic functions of commuting operator systems*, VINITI 857-79, 1-37 (Russian).

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