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ON A QUESTION OF HONG XUN YI

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ABSTRACT. In the paper we prove a uniqueness theorem for meromorphic functions which provides an answer to a question of H. X. Yi.

1. INTRODUCTION AND DEFINITIONS

Let f be a nonconstant meromorphic function defined on the open complex plane $\mathbb{C}$. Let S be a set of distinct complex numbers and $E_f(S) = \cup_{a\in S}\{z : f(z) - a = 0\}$, where a zero of $f - a$ of multiplicity m is repeated m times in $E_f(S)$.

Gross [3] proved that there exist three finite sets $S_j (j = 1, 2, 3)$ such that any two entire functions f and g satisfying $E_f(S_j) = E_g(S_j)$ for $j = 1, 2, 3$ must be identical.

For meromorphic functions Yi [11, 12] proved the following two theorems.

Theorem A [11]. *Let $S_1 = \{z : z^n - 1 = 0\}$, $S_2 = \{a, b\}$, $S_3 = \{\infty\}$, where $n(\geq 7)$ be a positive integer, a and b be constants such that $ab \neq 0$, $a^n \neq b^n$, $a^{2n} \neq 1$, $b^n \neq 1$ and $a^n b^n \neq 1$. If f and g are nonconstant meromorphic functions satisfying $E_f(S_j) = E_g(S_j)$ for $j = 1, 2, 3$ then $f \equiv g$.*

Theorem B [12]. *Let $S = \{z : z^n + az^{n-m} + b = 0\}$, where n and m are two positive integers such that $m \geq 2$, $n \geq 2m + 7$ with n and m having no common factor, a and b be two nonzero constants such that $z^n + az^{n-m} + b = 0$ has no multiple root. If f and g are nonconstant meromorphic functions satisfying $E_f(S) = E_g(S)$ and $E_f(\{\infty\}) = E_g(\{\infty\})$ then $f \equiv g$.*

One may note that the range set S in Theorem B contains at least eleven elements which corresponds to $m = 2$.

In [12] Yi asked the following question: "What can be said if $m = 1$ in Theorem B?"

To answer this question Yi [12] proved the following theorem.

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Theorem C [12]. *Let $S = \{z : z^n + az^{n-1} + b = 0\}$, where $n(\geq 9)$ be a positive integer and a, b be two nonzero constants such that $z^n + az^{n-1} + b = 0$ has no multiple root. If f and g are two nonconstant meromorphic functions such that $E_f(S) = E_g(S)$ and $E_f(\{\infty\}) = E_g(\{\infty\})$ then either $f \equiv g$ or*

$$f \equiv -\frac{aH(H^{n-1}-1)}{H^n-1} \quad \text{and} \quad g \equiv -\frac{a(H^{n-1}-1)}{H^n-1},$$

where H is a nonconstant meromorphic function.

Since one can verify that [12] $H \equiv f/g$, Theorem C is not much significant.

Lahiri [5] proved the following result which provides an answer to the question of Yi.

Theorem D [5]. *Let $S = \{z : z^n + az^{n-1} + b = 0\}$, where $n(\geq 8)$ be a positive integer and a, b be two nonzero constants such that $z^n + az^{n-1} + b = 0$ has no multiple root. If f and g are two nonconstant meromorphic functions having no simple pole such that $E_f(S) = E_g(S)$ and $E_f(\{\infty\}) = E_g(\{\infty\})$ then $f \equiv g$.*

Recently Fang and Lahiri [2] improved Theorem D and proved the following result.

Theorem E [2]. *Let $S = \{z : z^n + az^{n-1} + b = 0\}$, where $n(\geq 7)$ be a positive integer and a, b be two nonzero constants such that $z^n + az^{n-1} + b = 0$ has no multiple root. If f and g are two nonconstant meromorphic functions having no simple pole such that $E_f(S) = E_g(S)$ and $E_f(\{\infty\}) = E_g(\{\infty\})$ then $f \equiv g$.*

Considering $S = \{z : z^7 - z^6 - 1 = 0\}$ and

$$f = \frac{e^z + e^{2z} + \cdots + e^{6z}}{1 + e^z + e^{2z} + \cdots + e^{6z}} \quad \text{and} \quad g = \frac{1 + e^z + e^{2z} + \cdots + e^{5z}}{1 + e^z + e^{2z} + \cdots + e^{6z}}$$

it is verified that for the validity of Theorem E f and g must not have any simple pole. We further note that for these functions $\Theta(\infty; f) = \Theta(\infty; g) = 0$.

If two functions f and g have no simple pole then clearly $\Theta(\infty; f) + \Theta(\infty; g) \geq 1$. In the paper we show that if $\Theta(\infty; f) + \Theta(\infty; g) > 1$ then Theorem E remains valid even if f and g posses simple poles. Also we relax the nature of sharing the sets in Theorem E. To this end we explain the notion of weighted sharing as introduced in [6, 7].

Definition 1. [6, 7] Let k be a nonnegative integer or infinity. For $a \in C \cup \{\infty\}$ we denote by $E_k(a; f)$ the set of all a-points of f where an a-point of multiplicity m is counted m times if $m \leq k$ and $k + 1$ times if $m > k$. If $E_k(a; f) = E_k(a; g)$, we say that f, g share the value a with weight k.

The definition implies that if f, g share a value a with weight k then z_o is a zero of $f - a$ with multiplicity $m(\leq k)$ if and only if it is a zero of $g - a$ with multiplicity $m(\leq k)$ and z_o is a zero of $f - a$ with multiplicity $m(> k)$ if and only if it is a zero of $g - a$ with multiplicity $n(> k)$ where m is not necessarily equal to n.

We write f, g share (a,k) to mean that f, g share the value a with weight k. Clearly if f, g share (a,k) then f, g share (a,p) for all integer p, $0 \le p < k$. Also we note that f, g share a value a IM or CM if and only if f, g share $(a,0)$ or (a,∞) respectively.

Definition 2. [6] For $S \subset \mathbb{C} \cup \{\infty\}$, we define $E_f(S,k)$ as $E_f(S,k) = \cup_{a \in S} E_k(a;f)$, where k is a nonnegative integer or infinity.

Clearly $E_f(S) = E_f(S,\infty)$.

Definition 3. [6] If s is a positive integer, we denote by $N(r,a;f \mid= s)$ the counting function of those a-points of f whose multiplicity is s, where each a-point is counted according to its multiplicity.

Definition 4. [6] If s is a positive integer, we denote by $\overline{N}(r,a;f \mid\ge s)$ the counting function of those a-points of f whose multiplicities are greater than or equal to s, where each a-point is counted only once.

Definition 5. [1, 6, 8] If s is a nonnegative integer, we denote by $N_s(r,a;f)$ the counting function of a-points of f where an a-point with multiplicity m is counted m times if $m \le s$ and s times if $m > s$.

We put $N_\infty(r,a;f) \equiv N(r,a;f)$.

Definition 6. [6] Let f, g share a value a IM. We denote by $\overline{N}_*(r,a;f,g)$ the counting function of those a-points of f whose multiplicities are different from multiplicities of the corresponding a-points of g, where each a-point is counted only once.

Clearly $\overline{N}_*(r,a;f,g) \equiv \overline{N}_*(r,a;g,f)$.

In the paper we do not explain the standard notations and definitions of the value distribution theory as those are available in [4, 10]. Unless otherwise stated throughout the paper we denote by f, g two nonconstant meromorphic functions.

Following is the main result of the paper which provides an answer of the question of Yi [12].

Theorem 1. *Let $S = \{z : z^n + az^{n-1} + b = 0\}$, where $n(\ge 7)$ be a positive integer and a, b be two nonzero constants such that $z^n + az^{n-1} + b = 0$ has no multiple root. If $\Theta(\infty;f) + \Theta(\infty;g) > 1$ and $E_f(S,2) = E_g(S,2)$, $E_f(\{\infty\},\infty) = E_g(\{\infty\},\infty)$ then $f \equiv g$.*

2. Lemmas

In this section we discuss some lemmas which will be required in the sequel. Also we denote by H a meromorphic function defined as follows

$$H = \left(\frac{f''}{f'} - \frac{2f'}{f-1}\right) - \left(\frac{g''}{g'} - \frac{2g'}{g-1}\right).$$

Lemma 1. *If f, g share $(1,1)$ and $H \not\equiv 0$ then*

$$N(r,1;f \mid= 1) = N(r,1;g \mid= 1) \le N(r,H) + S(r,f) + S(r,g).$$

Proof. Since f, g share $(1,1)$, it follows that a simple 1-point of f is a simple 1-point of g and conversely. Let z_o be a simple 1-point of f and g. Then in some neighbourhood on z_o we get by a simple calculation

$$H(z) = (z - z_o)\phi(z)\,,$$

where ϕ is analytic at z_o.

Hence by the first fundamental theorem and Milloux theorem ([4], p. 55) we get

$$N(r,1;f\mid=1) \le N(r,0;H) \le N(r,H) + S(r,f) + S(r,g)\,,$$

from which the lemma follows because $N(r,1;f\mid=1) = N(r,1;g\mid=1)$. This proves the lemma. □

Lemma 2. *Let f, g share $(1,0)$, (∞,∞) and $H \not\equiv 0$. Then*

$$\begin{aligned} N(r,H) \le{}& \overline{N}(r,0;f\mid\ge 2) + \overline{N}(r,0;g\mid\ge 2) + \overline{N}_*(r,1;f,g) \\ &+ N_o(r,0;f') + \overline{N}_o(r,0;g')\,, \end{aligned}$$

where $\overline{N}_o(r,0;f')$ is the reduced counting function of those zeros of f' which are not the zeros of $f(f-1)$ and $\overline{N}_o(r,0;g')$ is similarly defined.

Proof. One can easily verify that possible poles of H occur at (i) multiple zeros of f, g; (ii) zeros of $f-1$, $g-1$; (iii) zeros of f' which are not the zeros of $f(f-1)$; and (iv) zeros of g' which are not the zeros of $g(g-1)$.

Let z_o be a zero of $f-1$ and $g-1$ with multiplicities m and n respectively. Then in some neighbourhood of z_o we get

$$H(z) = \frac{(m-n)\phi(z)}{z - z_o} + \psi(z)\,,$$

where ϕ, ψ are analytic at z_o and $\phi(z_o) \ne 0$.

This shows that if $m = n$ then z_o is not a pole of H and if $m \ne n$ then z_o is a simple pole of H. Since all the poles of H are simple, the lemma is proved. □

Lemma 3. *If f, g share $(1,2)$ then*

$$\begin{aligned} &\overline{N}_o(r,0;g') + \overline{N}(r,1;g\mid\ge 2) + \overline{N}_*(r,1;f,g) \\ &\quad\le \overline{N}(r,\infty;g) + \overline{N}(r,0;g) + S(r,g)\,. \end{aligned}$$

Proof. Remembering the definition of $\overline{N}_o(r,0;g')$ and noting that $\overline{N}_*(r,1;f,g) \le \overline{N}(r,1;g\mid\ge 3)$ because f, g share (1,2), we get

$$\begin{aligned} (1)\qquad &\overline{N}_o(r,0;g') + \overline{N}(r,1;g\mid\ge 2) + \overline{N}_*(r,1;f,g) + N(r,0;g) - \overline{N}(r,0;g) \\ &\le \overline{N}_o(r,0;g') + \overline{N}(r,1;g\mid\ge 2) + \overline{N}(r,1;g\mid\ge 3) \\ &\quad + N(r,0;g) - \overline{N}(r,0;g) \\ &\le N(r,0;g')\,. \end{aligned}$$

By the first fundamental theorem and Milloux theorem ([4], p. 55)

$$
\begin{aligned}
(2) \qquad N(r,0;g') &\le N(r,0;\frac{g'}{g}) + N(r,0;g) - \overline{N}(r,0;g) \\
&\le N(r,\frac{g'}{g}) + N(r,0;g) - \overline{N}(r,0;g) + S(r,g) \\
&= \overline{N}(r,\infty;g) + \overline{N}(r,0;g) + N(r,0;g) - \overline{N}(r,0;g) + S(r,g) \\
&= \overline{N}(r,\infty;g) + N(r,0;g) + S(r,g)\,.
\end{aligned}
$$

Now the lemma follows from (1) and (2). This proves the lemma. □

Lemma 4. [9] *Let* $P(f) = \sum_{j=0}^{n} a_j f^j$, *where* $a_o, a_1, \dots, a_n (\not\equiv 0)$ *are such that* $T(r,a_j) = S(r,f)$ *for* $j = 0,1,2,\dots,n$. *Then*

$$T(r,P(f)) = nT(r,f) + S(r,f)\,.$$

Lemma 5. *If* f, g *share* $(\infty,0)$ *then for* $n \ge 2$

$$f^{n-1}(f+a)g^{n-1}(g+a) \not\equiv b^2\,,$$

where a, b *are finite nonzero numbers.*

Proof. If possible let

$$f^{n-1}(f+a)g^{n-1}(g+a) \equiv b^2\,. \tag{3}$$

If f and g have no pole, from (3) it follows that f has no zero and $-a$-point, which is impossible.

If z_o is a pole of f, by (3) it follows that z_o is either a zero or an $-a$-point of g and this contradicts the fact that f, g share $(\infty,0)$. This proves the lemma. □

Lemma 6. *If* $\Theta(\infty;f) + \Theta(\infty;g) > 1$ *then for* $n \ge 6$

$$f^{n-1}(f+a) \equiv g^{n-1}(g+a)$$

implies $f \equiv g$, *where* a *is a finite nonzero number.*

Proof. Let

$$f^{n-1}(f-1) \equiv g^{n-1}(g-1)\,. \tag{4}$$

and suppose $f \not\equiv g$. We consider two cases:

(a) $y = g/f$ is a constant. Then from (4) it follows that $y \ne 1$, $y^{n-1} \ne 1$, $y^n \ne 1$ and

$$f \equiv -a\frac{1-y^{n-1}}{1-y^n} = \text{constant},$$

which leads to a contradiction.

(b) $y = g/f$ is not a constant. We can rewrite $f \equiv -a\frac{1-y^{n-1}}{1-y^n}$ in the form

$$f \equiv a\left(\frac{y^{n-1}}{1+y+y^2+\cdots+y^{n-1}} - 1\right)\,. \tag{5}$$

From (5) we get by the first fundamental theorem and Lemma 4

$$\begin{aligned} T(r,f) &= T(r,\sum_{j=0}^{n-1}\frac{1}{y^j}) + S(r,y) \\ &= (n-1)T(r,\frac{1}{y}) + S(r,y) \\ &= (n-1)T(r,y) + S(r,y)\,. \end{aligned}$$

Now we note that any pole of y does not contribute any pole of $\{y^{n-1}/\sum_{j=1}^{n-1}y^j\}-1$. So from (5) it follows that

$$\sum_{k=1}^{n-1}\overline{N}(r,u_k;y) \le \overline{N}(r,\infty;f)\,,$$

where $u_k = \exp(\frac{2k\pi i}{n})$, for $k=1,2,\dots,n-1$.

By the second fundamental theorem we get

$$\begin{aligned} (n-3)T(r,y) &\le \sum_{k=1}^{n-1}\overline{N}(r,u_k;y) + S(r,y) \\ &\le \overline{N}(r,\infty;f) + S(r,y) \\ &< (1-\Theta(\infty;f)+\varepsilon)T(r,f) + S(r,y) \\ &= (n-1)(1-\Theta(\infty;f)+\varepsilon)T(r,y) + S(r,y)\,, \end{aligned} \tag{6}$$

where $\varepsilon(>0)$.

Again putting $y_1 = \frac{1}{y}$, noting that $T(r,y) = T(r,y_1) + O(1)$ and proceeding as above we get

$$(n-3)T(r,y) \le (n-1)(1-\Theta(\infty;g)+\varepsilon)T(r,y) + S(r,y)\,, \tag{7}$$

where $\varepsilon(>0)$.

From (6) and (7) we get in view of the given condition

$$\begin{aligned} 2(n-3)T(r,y) &\le (n-1)(2-\Theta(\infty;f)-\Theta(\infty;g)+2\varepsilon)T(r,y) + S(r,y) \\ &< (n-1)(1+2\varepsilon)T(r,y) + S(r,y)\,, \end{aligned}$$

which implies a contradiction for all sufficiently small $\varepsilon(>0)$ because $n \ge 6$.

Hence $f \equiv g$ and this completes the proof of the lemma. □

3. Proof of Theorem 1

Let $F = -\frac{1}{b}f^{n-1}(f+a)$ and $G = -\frac{1}{b}g^{n-1}(g+a)$. We first show that following inequality does not hold:

$$\begin{aligned} T(r) \le\ & N_2(r,0;F) + N_2(r,0;G) + \overline{N}(r,\infty;F) + \overline{N}(r,\infty;G) \\ & + S(r,F) + S(r,G)\,, \end{aligned} \tag{8}$$

where $T(r) = \max\{T(r,F), T(r,G)\}$.

By Lemma 4 we see that

$$T(r,F) = nT(r,f) + S(r,f) \quad \text{and} \quad T(r,G) = nT(r,g) + S(r,g)\,. \tag{9}$$

Now

$$\begin{aligned} &N_2(r,0;F) + N_2(r,0;G) + \overline{N}(r,\infty;F) + \overline{N}(r,\infty;G) + S(r,F) + S(r,G) \\ &\qquad \le 2\overline{N}(r,0;f) + N_2(r,0;f+a) + 2\overline{N}(r,0;g) + N_2(r,0;g+a) \\ &\qquad\quad + \overline{N}(r,\infty;f) + \overline{N}(r,\infty;g) + S(r,F) + S(r,G) \\ &\qquad < 3T(r,f) + 3T(r,g) + \{1 - \Theta(\infty;f) + \varepsilon\}T(r,f) \\ &\qquad\quad + \{1 - \Theta(\infty;g) + \varepsilon\}T(r,g) + S(r,F) + S(r,G)\,, \end{aligned}$$

where $\varepsilon(>0)$ is given.

In view of (9) and the given condition we get

$$\begin{aligned} &N_2(r,0;F) + N_2(r,0;G) + \overline{N}(r,\infty;F) + \overline{N}(r,\infty;G) + S(r,F) + S(r,G) \\ &\qquad < \frac{1}{7}\{8 - \Theta(\infty;f) - \Theta(\infty;g) + 2\varepsilon\}T(r) + S(r,F) + S(r,G) \\ &\qquad = (1-\alpha)T(r) + S(r,F) + S(r,G)\,, \end{aligned}$$

where $\alpha(>0)$ and $\varepsilon(>0)$ are so chosen that $7\alpha = \Theta(\infty;f) + \Theta(\infty;g) - 1 - 2\varepsilon > 0$. This shows that (8) does not hold. Let

$$H = \left(\frac{F''}{F'} - \frac{2F'}{F-1}\right) - \left(\frac{G''}{G'} - \frac{2G'}{G-1}\right)\,.$$

We note that F, G share $(1,2)$ and (∞,∞) because $E_f(S,2) = E_g(S,2)$ and $E_f(\{\infty\},\infty) = E_g(\{\infty\},\infty)$.

Let $H \not\equiv 0$. Then by Lemma 1, Lemma 2 and Lemma 3 we obtain

$$\begin{aligned} N(r,1;F\,|=1) &\le \overline{N}(r,0;F\,|\ge 2) + \overline{N}(r,0;G\,|\ge 2) + \overline{N}(r,\infty;G) \\ &\quad + \overline{N}(r,0;G) - \overline{N}(r,1;G\,|\ge 2) + \overline{N}_o(r,0;F') + S(r,G)\,. \end{aligned} \tag{10}$$

By the second fundamental theorem we get

$$\begin{aligned} T(r,F) &\le \overline{N}(r,\infty;F) + \overline{N}(r,0;F) + \overline{N}(r,1;F) \\ &\quad - N(r,0;F') + S(r,F)\,. \end{aligned} \tag{11}$$

Since F, G share $(1,2)$ we see that

$$\begin{aligned} \overline{N}(r,1;F) &= N(r,1;F\,|=1) + \overline{N}(r,1;F\,|\ge 2) \\ &= N(r,1;F\,|=1) + \overline{N}(r,1;G\,|\ge 2)\,. \end{aligned} \tag{12}$$

From (10), (11) and (12) we get

$$\begin{aligned} T(r,F) &\le N_2(r,0;F) + N_2(r,0;G) + \overline{N}(r,\infty;F) \\ &\quad + \overline{N}(r,\infty;G) + S(r,F) + S(r,G)\,. \end{aligned} \tag{13}$$

Similarly we obtain

$$\begin{aligned} T(r,G) &\le N_2(r,0;F) + N_2(r,0;G) + \overline{N}(r,\infty;F) \\ &\quad + \overline{N}(r,\infty;G) + S(r,F) + S(r,G)\,. \end{aligned} \tag{14}$$

We see that (13) and (14) together imply (8) which does not hold. Hence $H \equiv 0$ and so

$$\frac{F''}{F'} - \frac{2F'}{F-1} \equiv \frac{G''}{G'} - \frac{2G'}{G-1}$$

i.e.,

$$(\log F')' - (2\log(F-1))' \equiv (\log G')' - (2\log(G-1))' .$$

From this equation we get

$$F \equiv \frac{AG+B}{CG+D}, \tag{15}$$

where A,B,C,D are complex numbers such that $AD - BC \neq 0$.

From (15) it follows that

$$T(r,F) = T(r,G) + O(1) . \tag{16}$$

We now consider the following cases.

Case I Let $AC \neq 0$. Then

$$F - \frac{A}{C} \equiv \frac{B - \frac{AD}{C}}{CG+D}$$

and so by the second fundamental theorem we get

$$\begin{aligned} T(r,F) &\leq \overline{N}(r,0;F) + \overline{N}(r,\infty;F) + \overline{N}(r,A/C;F) + S(r,F) \\ &= \overline{N}(r,0;F) + \overline{N}(r,\infty;F) + \overline{N}(r,\infty;G) + S(r,F) . \end{aligned}$$

This by (16) implies (8) which does not hold.

Case II Let $AC = 0$. Since $AD - BC \neq 0$, it follows that A and C are not simultaneously zero.

Let $A = 0$. Then from (15) we get

$$G + \frac{D}{C} \equiv \frac{B}{CF}, \tag{17}$$

where $BC \neq 0$.

If $D \neq 0$, from (17) we get by the second fundamental theorem

$$\begin{aligned} T(r,G) &\leq \overline{N}(r,0;G) + \overline{N}(r,\infty;F) + \overline{N}(r,-D/C;G) + S(r,G) \\ &= \overline{N}(r,0;G) + \overline{N}(r,\infty;G) + \overline{N}(r,\infty;F) + S(r,G) . \end{aligned}$$

This by (16) implies (8) which does not hold.

Let $D = 0$. Then from (17) we get

$$FG \equiv \frac{B}{C} . \tag{18}$$

Since F, G share (∞,∞), it follows from (18) that F has no zero and pole. Hence there exists $z_o \in \mathbb{C}$ such that $F(z_o) = G(z_o) = 1$ because F, G share $(1,2)$. So from (18) we get $\frac{B}{C} = 1$ and so $FG \equiv 1$ i.e.

$$f^{n-1}(f+a)g^{n-1}(g+a) \equiv b^2$$

which is impossible by Lemma 5.

Let $C = 0$. Then from (15) we get

$$F \equiv \frac{A}{D}G + \frac{B}{D}, \tag{19}$$

where $AD \neq 0$.

If $B \neq 0$, from (19) we get by the second fundamental theorem

$$\begin{aligned} T(r,F) &\leq \overline{N}(r,0;F) + \overline{N}(r,\infty;F) + \overline{N}(r,B/D;F) + S(r,F) \\ &= \overline{N}(r,0;F) + \overline{N}(r,\infty;F) + \overline{N}(r,0;G) + S(r,F). \end{aligned}$$

This by (16) implies (8) which does not hold.

Let $B = 0$. Then from (19) we get

$$F \equiv \frac{AG}{D}. \tag{20}$$

If F has no 1-point, by the second fundamental theorem we get

$$T(r,F) \leq \overline{N}(r,0;F) + \overline{N}(r,\infty;F) + S(r,F).$$

This by (16) implies (8) which does not hold.

Let $F(z_o) = 1$ for some $z_o \in \mathbb{C}$. Since F, G share $(1,2)$, we get $G(z_o) = 1$ and so from (20) it follows that $\frac{A}{D} = 1$. Therefore $F \equiv G$ i.e.

$$f^{n-1}(f+a) \equiv g^{n-1}(g+a)$$

which implies by Lemma 6 that $f \equiv g$. This proves the theorem. □

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