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ANSWER TO ONE OF FISHBURN'S QUESTIONS (NOTE)

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Abstract. Our short note gives the affirmative answer to one of Fishburn's questions.

A finite set is k-isosceles for $k \geq 3$ if every k-point subset of the set contains a point equidistant from two others. P. Fishburn has formulated six open questions in the conclusion of his interesting paper [1], the first of which is: "Is there a 6-point F with no 4 points on a circle and no 3 points on a line?" (F denotes a 4-isosceles planar set).

Result. The answer to the above question is affirmative.

Proof. In the Fig.1 the points P_1, P_2, P_3 lie on the circle with centre P_4, i.e. $|P_4P_1| = |P_4P_2| = |P_4P_3|$. Moreover, we take these points such that $|P_3P_1| = |P_3P_2|$. Let us denote by $m_{i,j}$ the midperpendicular of the line segments P_iP_j and let us take $P_5 = m_{3,4} \cap m_{1,4}$. Hence $|P_5P_1| = |P_5P_4| = |P_5P_3|$, and so $P_5 \in m_{1,3}$. Now it is sufficient to take the point $P_6 \in m_{2,5}$ "almost anywhere", more precisely with the exception of the finite number of points of $m_{2,5}$ (e.g. $P_6 \notin P_2P_5, P_6 \notin P_2P_3, \dots$).

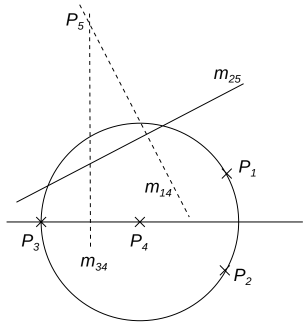

Fig.1

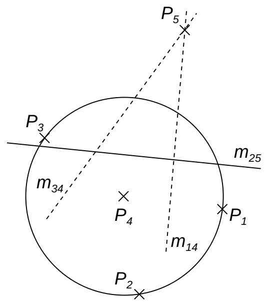

Fig.2

Remark. If the points P_1, P_2, P_3 are vertices of a regular pentagon, as it is in the next Fig.2, and if in the construction before we take $P_6 \in m_{2,5} \cap m_{2,4}$, then

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we obtain a slightly better configuration, because $P_6 \in m_{4,5}$, too. But neither to this hopeful configuration it is possible to add a point P_7 in a way to obtain 7-point F with no 4 points on a circle, and so affirmatively answer 2-nd Fishburn's question. From this construction we start to believe that the answer to Fishburn's 2-nd question will be NO.

References

[1] Fishburn, P., *Isosceles Planar Subsets*, Discrete & Computational Geometry **19** (1998), 391–398.

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