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THE NATURAL AFFINORS ON $(J^rT^*)^*$

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Abstract. For natural numbers r and $n \geq 2$ a complete classification of natural affinors on the natural bundle $(J^rT^*)^*$ dual to r-jet prolongation J^rT^* of the cotangent bundle over n-manifolds is given.

0. The r-jet prolongation J^rT^*M of the cotangent bundle T^*M of an n-manifold M is the space of all r-jets of 1-forms on M, i.e.

$$J^rT^*M = \{j^r_x\omega \mid \omega \text{ is a 1-form on } M\ ,\ x \in M\}\,.$$

It is a vector bundle over M with respect to the source projection. Let $(J^rT^*)^*M = (J^rT^*M)^*$ be the dual bundle and let $\pi : (J^rT^*)^*M \to M$ be its projection. Clearly, every embedding $\varphi : M \to N$ of two n-manifolds induces functorially (in obvious way) a vector bundle mapping $(J^rT^*)^*\varphi : (J^rT^*)^*M \to (J^rT^*)^*N$ over φ, and we obtain a natural vector bundle $(J^rT^*)^* : \mathcal{M}_n \to \mathcal{VB} \subset \mathcal{FM}$.

In general, a natural affinor A on a natural bundle $F : \mathcal{M}_n \to \mathcal{FM}$ is a system of affinors

$$A : TFM \to TFM$$

(i.e. tensor fields of type (1,1) on FM) for any n-manifold M which is invariant with respect to local embeddings between n-manifolds.

For example, the family $id = id_{TFM} : TFM \to TFM$ for any n-manifold M is a natural affinor on F.

Another example of a natural affinor on $(J^rT^*)^*$ is the family

$$\delta : T(J^rT^*)^*M \to (J^rT^*)^*M \times_M TM \subset (J^rT^*)^*M \times_M (J^rT^*)^*M \tilde{=} V(J^rT^*)^*M\,,$$

where the arrow is the system $(\pi^T, T\pi) : T(J^rT^*)^*M \to (J^rT^*)^*M \times_M TM$, $\pi^T : T(J^rT^*)^*M \to (J^rT^*)^*M$ is the tangent bundle projection, the inclusion $\subset$ is induced by the bundle map dual to the target projection $J^rT^*M \to T^*M$, $\tilde{=}$ is the standard canonical identification $E \times_M E \tilde{=} VE$ for any vector bundle $E \to M$, $VE \subset TE$ is the vertical bundle of E.

The main result of this note is the following classification theorem.

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Theorem 1. *If $n \geq 2$ and r are natural numbers, then any natural affinor A on $(J^rT^*)^*$ over n-manifolds is a linear combination (over $\mathbf{R}$) of id and δ.*

In [7], we proved that if r and $n \geq 2$ are natural numbers, then any natural affinor A on J^rT, the r-jet prolongation of the tangent bundle T, is proportional to the identity affinor. Then (as a corollary of Theorem 1) the natural bundles J^rT and $(J^rT^*)^*$ are not naturally isomorphic for r and n as above.

Natural affinors on F play a very improtrant role in the differential geometry. For example, they can be used to define torsions of a connection on F, see [5]. That is why classifications of natural affinors on some natural bundles has been studied in many papers, see e.g. [1]-[3] and [6]-[8].

Throughout this note the usual coordinates on $\mathbf{R}^n$ are denoted by $x^1, \dots, x^n$ and $\partial_i = \frac{\partial}{\partial x^i}$, $i = 1, \dots, n$.

All manifolds and maps are assumed to be of class C^∞.

1. We have a linear natural transformation $\tilde{\delta} : T(J^rT^*)^* \to (J^rT^*)^*$ given by

$$\tilde{\delta} : T(J^rT^*)^*M \to TM \subset (J^rT^*)^*M$$

for any n-manifold M, where the arrow is $T\pi : T(J^rT^*)^*M \to TM$ and the inclusion $TM \subset (J^rT^*)^*M$ is defined in Item 0. The linearity of $\tilde{\delta}$ means that $\tilde{\delta}$ induces a linear map $T_y(J^rT^*)^*M \to (J^rT^*)^*_{\pi(y)}M$ for any $y \in (J^rT^*)^*M$.

The crucial point in the proof of Theorem 1 is the following proposition.

Proposition 1. *If $n \geq 2$ and r are natural numbers, then any linear natural transformation $A : T(J^rT^*)^* \to (J^rT^*)^*$ over n-manifolds is proportional (by a real number) to $\tilde{\delta}$.*

Proof. Clearly, any element from the fibre $(J^rT^*)^*_0\mathbf{R}^n$ is a linear combination of the $(j^r_0(x^\alpha dx^i))^*$ for all $\alpha \in (\mathbf{N} \cup \{0\})^n$ with $|\alpha| \leq r$ and $i = 1, \dots, n$, where the $(j^r_0(x^\alpha dx^i))^*$ form the basis dual to the $j^r_0(x^\alpha dx^i) \in (J^rT^*)_0\mathbf{R}^n$ for α and i as beside.

Any natural transformation A as in the proposition is uniquely determined by the values $\langle A(u), j^r_0(x^\alpha dx^i)\rangle \in \mathbf{R}$ for $u \in (T(J^rT^*)^*\mathbf{R}^n)_0 \tilde{=} \mathbf{R}^n \times (V(J^rT^*)^*\mathbf{R}^n)_0$ $\tilde{=}$ $\mathbf{R}^n \times (J^rT^*)^*_0\mathbf{R}^n \times (J^rT^*)^*_0\mathbf{R}^n$, $\alpha \in (\mathbf{N} \cup \{0\})^n$ with $|\alpha| \leq r$ and $i = 1, \dots, n$, where $\tilde{=}$ is the standard trivialization and the canonical identification.

Since A is invariant with respect to the coordinate permutations, it is uniquely determined by the $\langle A(u), j^r_0(x^\alpha dx^1)\rangle$ for any u and α as above.

If $|\alpha| \geq 1$, then the local diffeomorphisms $\varphi_\alpha = (x^1, x^2 + x^\alpha, x^3, \dots, x^n)^{-1}$ sends $j^r_0(x^2dx^1)$ into $j^r_0(x^2dx^1) + j^r_0(x^\alpha dx^1)$. Then (using the invariancy of A with respect to the φ's) A is uniquely determined by the $\langle A(u), j^r_0(x^2dx^1)\rangle \in \mathbf{R}$ and the $\langle A(u), j^r_0(dx^1)\rangle \in \mathbf{R}$ for any $u \in (T(J^rT^*)^*\mathbf{R}^n)_0 \tilde{=} \mathbf{R}^n \times (J^rT^*)^*_0\mathbf{R}^n \times (J^rT^*)^*_0\mathbf{R}^n$.

At first we study the $\langle A(u), j^r_0(dx^1)\rangle \in \mathbf{R}$ for u as above.

By the naturality of A with respect to the homotheties $a_t = (t^1x^1, \dots, t^nx^n)$ for $t = (t^1, \dots, t^n) \in \mathbf{R}^n_+$, $\langle A(T(J^rT^*)^*(a_t)(u)), j^r_0(dx^1)\rangle = t^1\langle A(u), j^r_0(dx^1)\rangle$ for any

$t=(t^1,\dots,t^n)\in\mathbf{R}^n_+$. For any $t\in\mathbf{R}^n$, any $i=1,\dots,n$ and any $\alpha\in(\mathbf{N}\cup\{0\})^n$ we have $T(J^rT^*)^*(a_t)((j^r_0(x^\alpha dx^i))^*)=t^{\alpha+e_i}(j^r_0(x^\alpha dx^i))^*$. Then by the homogeneous function theorem, see [4], we have

$$\langle A(u),j^r_0(dx^1)\rangle=\lambda u^1_1+\mu u_{2,(0),1}+\nu u_{3,(0),1} \tag{1.1}$$

for some $\lambda,\mu,\nu\in\mathbf{R}$, where $u=(u_1,u_2,u_3)$, $u_1=(u^1_1,\dots,u^n_1)\in\mathbf{R}^n$, $u_{2,\alpha,i}$ is the coefficient of $u_2\in(J^rT^*)^*_0\mathbf{R}^n$ corresponding to $(j^r_0(x^\alpha dx^i))^*$, and $u_{3,\alpha,i}$ is the coefficient of $u_3\in(J^rT^*)^*_0\mathbf{R}^n$ on $(j^r_0(x^\alpha dx^i))^*$, $(0)=(0,\dots,0)\in(\mathbf{N}\cup\{0\})^n$.

Since A is linear, $\langle A(u_1,u_2,u_3),j^r_0(dx^1)\rangle$ is linear in (u_1,u_3) for any u_2. Then $\mu=0$. Replacing A by $A-\lambda\tilde{\delta}$, we can assume that $\lambda=0$. Then

$$\langle A(\partial^C_1{}_{|w}),j^r_0(dx^1)\rangle=\langle A(e_1,w,0),j^r_0(dx^1)\rangle=0 \tag{1.2}$$

for $w\in(J^rT^*)^*_0\mathbf{R}^n$, where $(\ )^C$ is the complete lift of vector fields to $(J^rT^*)^*$.

We prove that $\nu=0$.

It is sufficient to show that $\langle A(0,0,(j^r_0(dx^1))^*),j^r_0(dx^1)\rangle=0$.

For showing the last equality we prove

$$\begin{aligned}0&=\langle A(((x^1)^{r+1}\partial_1)^C_{|w}),j^r_0(dx^1)\rangle\\&=(r+1)\langle A(0,w,(j^r_0(dx^1))^*),j^r_0(dx^1)\rangle\\&=(r+1)\langle A(0,0,(j^r_0(dx^1))^*),j^r_0(dx^1)\rangle\,,\end{aligned}\tag{1.3}$$

where $w=(j^r_0((x^1)^r dx^1))^*$.

The third equality of (1.3) is clear as in the formula (1.1) λ and μ are 0.

We can prove the first equality of (1.3) as follows. Vector fields $\partial_1+(x^1)^{r+1}\partial_1$ and ∂_1 have the same r-jets at 0. Then, by the result of Zajtz [9], there exists a diffeomorphism $\varphi:\mathbf{R}^n\to\mathbf{R}^n$ such that $j^{r+1}_0\varphi=id$ and $\varphi_*\partial_1=\partial_1+(x^1)^{r+1}\partial_1$ near 0. Clearly, φ preserves $j^r_0(dx^1)$ because of the jet argument. Then, using the naturality of A with respect to φ, from (1.2) it follows that $\langle A((\partial_1+(x^1)^{r+1}\partial_1)^C{}_{|w}),j^r_0(dx^1)\rangle=0$ for any $w\in(J^rT^*)^*_0\mathbf{R}^n$. Now, applying the linearity of A, we end the proof of the first equality.

It remains to prove the second equality of (1.3). Let φ_t be the flow of $(x^1)^{r+1}\partial_1$. For any $\alpha\in(\mathbf{N}\cup\{0\})^n$ with $|\alpha|\le r$ we have

$$\begin{aligned}&\langle((x^1)^{r+1}\partial_1)^C_{|w},j^r_0(x^\alpha dx^i)\rangle\\&=\langle\frac{d}{dt}_{|t=0}(J^rT^*)^*_0(\varphi_t)(w),j^r_0(x^\alpha dx^i)\rangle\\&=\frac{d}{dt}_{|t=0}\langle(J^rT^*)^*_0(\varphi_t)(w),j^r_0(x^\alpha dx^i)\rangle\\&=\frac{d}{dt}_{|t=0}\langle w,j^r_0((\varphi_{-t})_*(x^\alpha dx^i))\rangle\\&=\langle w,j^r_0(\frac{d}{dt}_{|t=0}(\varphi_{-t})_*(x^\alpha dx^i))\rangle\\&=\langle w,j^r_0(L_{(x^1)^{r+1}\partial_1}(x^\alpha dx^i))\rangle\\&=\langle w,j^r_0(\alpha_1(x^1)^r x^\alpha dx^i+(r+1)\delta^i_1(x^1)^r x^\alpha dx^1)\rangle\,.\end{aligned}$$

Because of the definition of w, the last term is equal to $r+1$ if $j^r_0(x^\alpha dx^i) = j^r_0(dx^1)$ and it is equal to 0 in the other cases. Then $((x^1)^{r+1}\partial_1)^C_{|w} = (r+1)(j^r_0(dx^1))^*$ under the isomorphism $V_w((J^rT^*)^*\mathbf{R}^n)\tilde{=}(J^rT^*)^*_0\mathbf{R}^n$. It implies the second equality of (1.3).

To end the proof of the proposition it remains to show $\langle A(u), j^r_0(x^2dx^1)\rangle = 0$ for any $u \in (T(J^rT^*)^*\mathbf{R}^n)_0\tilde{=}\mathbf{R}^n \times (J^rT^*)^*_0\mathbf{R}^n \times (J^rT^*)^*_0\mathbf{R}^n$.

To prove this we use similar procedure as in the case of $\langle A(u), j^r_0(dx^1)\rangle$.

By the naturality of A with respect to the homotheties $a_t = (t^1x^1, \dots, t^nx^n)$ for $t = (t^1, \dots, t^n) \in \mathbf{R}^n_+$, the homogeneous function theorem and the linearity of A, one can easily deduce

$$\begin{aligned}\langle A(u), j^r_0(x^2dx^1)\rangle &= \lambda u_{3,e_2,1} + \mu u_{3,e_1,2} + \nu u^1_1 u_{2,(0),2} \\ &\quad + \rho u^2_1 u_{2,(0),1} + \sigma u_{2,(0),1}u_{3,(0),2} + \kappa u_{2,(0),2}u_{3,(0),1}\end{aligned} \tag{1.4}$$

for some $\lambda, \mu, \nu, \rho, \sigma, \kappa \in \mathbf{R}$, where $u = (u_1, u_2, u_3)$, $u_1 = (u^1_1, \dots, u^n_1) \in \mathbf{R}^n$, $u_2, u_3 \in (J^rT^*)^*_0\mathbf{R}^n$, and $u_{\tau,\alpha,i}$ is the coefficient of u_τ on $(j^r_0(x^\alpha dx^i))^*$, $\tau \in \{2, 3\}$, $e_i = (0, \dots, 1, 0, \dots, 0) \in (\mathbf{N} \cup \{0\})^n$, 1 in i-position.

Then

$$\langle A(\partial^C_1{}_{|w}), j^r_0(x^2dx^1)\rangle = 0 \tag{1.5}$$

for any linear combination w of the $(j^r_0(x^\alpha dx^i))^* \in (J^rT^*)^*_0\mathbf{R}^n$ for $|\alpha| \geq 1$ and $i = 1, \dots, n$.

At first we prove that $\lambda = \mu = 0$, i.e. $\langle A(0, 0, (j^r_0(x^2dx^1))^*), j^r_0(x^2dx^1)\rangle = 0$ and $\langle A(0, 0, (j^r_0(x^1dx^2))^*), j^r_0(x^2dx^1)\rangle = 0$.

For showing the equality $\langle A(0, 0, (j^r_0(x^2dx^1))^*), j^r_0(x^2dx^1)\rangle = 0$ we prove

$$\begin{aligned} 0 &= \langle A(((x^1)^r\partial_1)^C_{|w}), j^r_0(x^2dx^1)\rangle \\ &= r\langle A(0, w, (j^r_0(x^2dx^1))^*), j^r_0(x^2dx^1)\rangle \\ &= r\langle A(0, 0, (j^r_0(x^2dx^1))^*), j^r_0(x^2dx^1)\rangle\,, \end{aligned} \tag{1.6}$$

where $w = (j^r_0(x^2(x^1)^{r-1}dx^1))^* \in (J^rT^*)^*_0\mathbf{R}^n$.

The third equality of (1.6) is clear, see (1.4).

We can prove the first equality of (1.6) as follows. Vector fields $\partial_1 + (x^1)^r\partial_1$ and ∂_1 have the same $r-1$-jets at 0. Then, by the result of Zajtz, there exists a diffeomorphism $\varphi = \varphi_1 \times id_{\mathbf{R}^{n-1}} : \mathbf{R}^n = \mathbf{R} \times \mathbf{R}^{n-1} \to \mathbf{R}^n = \mathbf{R} \times \mathbf{R}^{n-1}$ such that $\varphi_1 : \mathbf{R} \to \mathbf{R}$, $j^r_0\varphi = id$ and $\varphi_*\partial_1 = \partial_1 + (x^1)^r\partial_1$ near 0. Let φ^{-1} send w into $\tilde{w}$. Then $\tilde{w}$ is the linear combination of the $(j^r_0(x^\alpha dx^i))^* \in (J^rT^*)^*_0\mathbf{R}^n$ for $|\alpha| \geq 1$ and $i = 1, \dots, n$. (For, $\langle \tilde{w}, j^r_0(dx^j)\rangle = \langle w, j^r_0(d(x^j \circ \varphi^{-1}))\rangle = 0$ for $j = 1, \dots, n$.) Consequently $\langle A(\partial^C_1{}_{|\tilde{w}}), j^r_0(x^2dx^1)\rangle = 0$. Clearly, φ preserves $j^r_0(x^2dx^1)$. Then, using the naturality of A with respect to φ it follows that $\langle A((\partial_1 + (x^1)^r\partial_1)^C{}_{|w}), j^r_0(x^2dx^1)\rangle = 0$. Now, applying the linearity of A, we end the proof of the first equality.

It remains to prove the second equality of (1.6). Using the flow argument one can easily compute

$$\langle ((x^1)^r\partial_1)^C_{|w}, j^r_0(x^\alpha dx^i)\rangle = \langle w, j^r_0(\alpha_1(x^1)^{r-1}x^\alpha dx^i + r\delta^i_1(x^1)^{r-1}x^\alpha dx^1)\rangle .$$

Because of the definition of w, the last term is equal to r if $j^r_0(x^\alpha dx^i) = j^r_0(x^2dx^1)$ and it is equal to 0 in the other cases. Then $((x^1)^r\partial_1)^C_{|w} = r(j^r_0(x^2dx^1))^*$ under the isomorphism $V_w((J^rT^*)^*\mathbf{R}^n)\tilde{=}(J^rT^*)^*_0\mathbf{R}^n$. It implies the second equality of (1.6).

For showing the equality $\langle A(0,0,(j^r_0(x^1dx^2))^*), j^r_0(x^2dx^1)\rangle = 0$ we prove

$$\begin{aligned} 0 &= \langle A(((x^1)^r\partial_1)^C_{|w}), j^r_0(x^2dx^1)\rangle \\ &= \langle A(0,w,(j^r_0(x^1dx^2))^*), j^r_0(x^2dx^1)\rangle \\ &= \langle A(0,0,(j^r_0(x^1dx^2))^*), j^r_0(x^2dx^1)\rangle , \end{aligned} \tag{1.7}$$

where $w = (j^r_0((x^1)^rdx^2))^*$.

The third equality of (1.7) is clear, see (1.4).

The first equality of (1.7) has similar proof as the first equality of (1.6).

It remains to prove the second equality of (1.7). Using the flow argument one can easily compute $((x^1)^r\partial_1)^C_{|w} = (j^r_0(x^1dx^2))^*$ under the obvious isomorphism. It implies the second equality of (1.7).

We have proved that $\mu = \lambda = 0$.

Now, we prove that $\sigma = \kappa = 0$.

Local diffeomorphism $\psi = (x^1 + \frac{1}{2}(x^1)^2, \frac{x^2}{1+x^1}, x^3, \dots, x^n)$ preserves $j^r_0(x^2dx^1)$, it sends $(j^r_0(x^1dx^2))^*$ into $(j^r_0(x^1dx^2))^* - (j^r_0(dx^2))^*$, and it preserves $(j^r_0(dx^1))^*$. Now, using (1.4) with $\lambda = \mu = 0$ and the naturality of A with respect to ψ we obtain

$$\begin{aligned} 0 &= -\langle A(0,(j^r_0(dx^1))^*,(j^r_0(x^1dx^2))^*), j^r_0(x^2dx^1)\rangle \\ &= -\langle A(0,(j^r_0(dx^1))^*,(j^r_0(x^1dx^2))^*) - (j^r_0(dx^2))^*), j^r_0(x^2dx^1)\rangle \\ &= \langle A(0,(j^r_0(dx^1))^*,(j^r_0(dx^2))^*), j^r_0(x^2dx^1)\rangle . \end{aligned} \tag{1.8}$$

Therefore in (1.4) we have $\sigma = 0$.

Similarly, starting from $0 = -\langle A(0,(j^r_0(x^1dx^2))^*,(j^r_0(dx^1))^*), j^r_0(x^2dx^1)\rangle$ we obtain $\langle A(0,(j^r_0(dx^2))^*,(j^r_0(dx^1))^*), j^r_0(x^2dx^1)\rangle = 0$, i.e. $\kappa = 0$.

Now, we prove that in (1.4) we have $\nu = 0$.

The above local diffeomorphism ψ sends the germ at 0 of ∂_1 into the germ at 0 of $\partial_1 + \dots$, where the dots is some vector field vanishing in $0 \in \mathbf{R}^n$. Now, using (1.4) with $\lambda = \mu = \sigma = \kappa = 0$ and the naturality of A with respect to ψ we obtain

$$\begin{aligned} 0 &= -\langle A(e_1,(j^r_0(x^1dx^2))^*,0), j^r_0(x^2dx^1)\rangle \\ &= -\langle A(e_1,(j^r_0(x^1dx^2))^*) - (j^r_0(dx^2))^*, u_3), j^r_0(x^2dx^1)\rangle \\ &= \langle A(e_1,(j^r_0(dx^2))^*,0), j^r_0(x^2dx^1)\rangle \end{aligned} \tag{1.9}$$

for some $u_3 \in (J^rT^*)^*_0\mathbf{R}^n$. Therefore in (1.4) we have $\nu = 0$.

It remains to prove that in (1.4) we have $\rho = 0$.

From (1.4) with $\mu = \lambda = \nu = \sigma = \kappa = 0$ and the invariancy of A with respect to the diffeomorphism permuting x^1 and x^2 we have

$$\langle A(e_2, (j^r_0(dx^1))^*, u_3), j^r_0(x^1dx^2)\rangle = 0 \tag{1.10}$$

for any $u_3 \in (J^rT^*)^*_0\mathbf{R}^n$. From (1.1) with $\lambda = \mu = \nu = 0$ we have

$$\langle A(e_2, (j^r_0(dx^1))^*, 0), j^r_0(dx^1)\rangle = 0\,. \tag{1.11}$$

Now, using the invariancy of A with respect to $\Theta = (x^1 + x^1x^2, x^2, \dots, x^n)^{-1}$ from (1.11) we obtain

$$\begin{aligned}0 &= \langle A(e_2, (j^r_0(dx^1))^*, u_3), j^r_0(dx^1) + j^r_0(x^1dx^2) + j^r_0(x^2dx^1)\rangle\\ &= \langle A(e_2, (j^r_0(dx^1))^*, 0), j^r_0(x^2dx^1)\rangle\end{aligned}$$

for some u_3 because of (1.1) with $\lambda = \mu = \nu = 0$, (1.4) with $\lambda = \mu = \nu = \sigma = \kappa = 0$ and (1.10). Therefore in (1.4) we have $\rho = 0$.

The proof of Proposition 1 is complete. □

2. The tangent map $T\pi : T(J^rT^*) * M \to TM$ of the bundle projection $\pi : (J^rT^*)^*M \to M$ defines a linear natural transformation $T\pi : T(J^rT^*)^* \to T$ over n-manifolds.

Proposition 2. *If r and $n \geq 2$ are natural numbers, then any linear natural transformation $A : T(J^rT^*)^* \to T$ over n-manifolds is proportional (by a real number) to $T\pi$.*

Proof. Applying the inclusion $TM \subset (J^rT^*)^*M$, we have $A : T(J^rT^*)^*M \to TM \subset (J^rT^*)^*M$. Then by Proposition 1, $A : T(J^rT^*)^*M \to TM \subset (J^rT^*)^*M$ is proportional to $\tilde{\delta}$. Then $A : T(J^rT^*)^*M \to TM$ is proportional to $T\pi$. □

3. In Item 0, we defined natural affinor $\delta : T(J^rT^*)^*M \to (J^rT^*)^*M \times_M TM \subset (J^rT^*)^*M \times_M (J^rT^*)^*M \tilde{=} V(J^rT^*)^*M$.

Proposition 3. *If r and $n \geq 2$ are natural numbers, then any natural affinor $A : T(J^rT^*)^*M \to V(J^rT^*)^*M$ on $(J^rT^*)^*$ over n-manifolds is proportional (by a real number) to δ.*

Proof. Define a linear natural transformation $\tilde{A} = pr_2 \circ A : T(J^rT^*)^*M \to V(J^rT^*)^*M \tilde{=} (J^rT^*)^*M \times_M (J^rT^*)^*M \to (J^rT^*)^*M$, where $pr_2 : (J^rT^*)^*M \times_M (J^rT^*)^*M \to (J^rT^*)^*M$ is the projection onto second factor. By Proposition 1, $\tilde{A} = \lambda\tilde{\delta}$ for some $\lambda \in \mathbf{R}$. Then $A = (\pi^T, \tilde{A}) = \lambda(\pi^T, \tilde{\delta}) = \lambda\delta$. □

4. We are now in position to prove Theorem 1. Let $A : T(J^rT^*)^*M \to T(J^rT^*)^*M$ be the natural affinor on $(J^rT^*)^*$ over n-manifolds. Then $T\pi \circ A : T(J^rT^*)^* \to T$ is a linear natural transformation. By Proposition 2, $T\pi \circ A = \lambda T\pi$ for some λ. Clearly, $T\pi \circ id = T\pi$. Then $A - \lambda id$ is an affinor on $(J^rT^*)^*$ of vertical type. Now, applying Proposition 3 we end the proof. □

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