ARCHIVUM MATHEMATICUM (BRNO)
Tomus 36 (2000), 255 – 260

THE NATURAL OPERATORS LIFTING VECTOR FIELDS TO $(J^rT^*)^*$

WŁODZIMIERZ M. MIKULSKI

Abstract. For integers $r \geq 2$ and $n \geq 2$ a complete classification of all natural operators $A : T_{|\mathcal{M}_n} \rightsquigarrow T(J^rT^*)^*$ lifting vector fields to vector fields on the natural bundle $(J^rT^*)^*$ dual to r-jet prolongation J^rT^* of the cotangent bundle over n-manifolds is given.

0. The r-jet prolongation J^rT^*M of the cotangent bundle T^*M of an n-manifold M is the space of all r-jets of 1-forms on M, i.e. $J^rT^*M = \{j^r_x\omega \mid \omega$ is a 1-form on M , $x \in M\}$. It is a vector bundle over M with respect to the source projection. Let $\pi : (J^rT^*)^*M = (J^rT^*M)^* \to M$ be the dual vector bundle. Clearly, every embedding $\varphi : M \to N$ of two n-manifolds induces functorially (in obvious way) a vector bundle mapping $(J^rT^*)^*\varphi : (J^rT^*)^*M \to (J^rT^*)^*N$ over φ, and we obtain a natural vector bundle $(J^rT^*)^* : \mathcal{M}_n \to \mathcal{VB} \subset \mathcal{FM}$.

In this paper, we study the problem how a vector field X on an n-manifold M induces canonically a vector field $A(X)$ on $(J^rT^*)^*M$. This problem is reflected in the concept of natural operators $A : T_{|\mathcal{M}_n} \rightsquigarrow T(J^rT^*)^*$ in the sense of Kolář, Michor and Slovák [5]. We prove that if $n \geq 2$ and $r \geq 2$ are integers, then the set of all natural operators $A : T_{|\mathcal{M}_n} \rightsquigarrow T(J^rT^*)^*$ is a vector space over $\mathbf{R}$ of dimension $r + 3$. We construct explicitly a basis of this vector space.

Various natural operators lifting vector fields are used practically in all papers in which problems of prolongations of geometric structures have been studied. That is why classifications of all natural operators lifting vector fields to some natural bundles have been studied in papers [1]-[9] and many others.

Throughout this note the usual coordinates on $\mathbf{R}^n$ are denoted by $x^1, \dots, x^n$ and $\partial_i = \frac{\partial}{\partial x^i}$, $i = 1, \dots, n$.

All manifolds and maps are assumed to be of class C^∞.

1. Let X be a vector field on an n-manifold M.

Example 1. We have the complete lift $(J^rT^*)^*X$ of X to $(J^rT^*)^*M$.

1999 *Mathematics Subject Classification*: 58A20, 53A55.
Key words and phrases: bundle functors, natural transformations, natural operators.
Reveived September 20, 1999.

Example 2. We have the Liouville vector field L on $(J^rT^*)^*M$.

Example 3. If $s \in \{1, \dots, r+1\}$, we define a vertical vector field $A^{(s)}(X)$ on $(J^rT^*)^*M$, $A^{(s)}(X)_{|u} := (u, \tilde{A}^{(s)}(X)(x)) \in \{u\} \times (J^rT^*)^*_xM \tilde{=} V_u((J^rT^*)^*M)$, $u \in (J^rT^*)^*_xM$, $x \in M$, where $\tilde{A}^{(s)}(X)(x) : (J^rT^*)_xM \to \mathbf{R}$ is a linear map given by $\tilde{A}^{(s)}(X)(x)(j^r_x\omega) := X^{s-1}\omega(X)(x)$, $X^{s-1} = X \circ \dots \circ X$ ($(s-1)$-times of X), ω is a 1-form on M.

Thus for natural numbers r and n we have $r+3$ natural operators $T_{|\mathcal{M}_n} \rightsquigarrow T(J^rT^*)^*$. Namely, we have $(J^rT^*)^*$, L and $A^{(s)}$ for $s = 1, \dots, r+1$.

Clearly, given natural numbers r and n the set of all natural operators $T_{|\mathcal{M}_n} \rightsquigarrow T(J^rT^*)^*$ is a vector space over $\mathbf{R}$ with respect to the obvious operations.

The main result of this paper is the following classification theorem.

Theorem 1. *If $n \geq 2$ and $r \geq 2$ are integers, then the natural operators $(J^rT^*)^*$, L and $A^{(s)}$ for $s = 1, \dots, r+1$ form a basis over $\mathbf{R}$ of the vector space of all natural operators $T_{|\mathcal{M}_n} \rightsquigarrow T(J^rT^*)^*$.*

The proof of Theorem 1 will occupy the rest of this paper.

2. In this item we study natural transformations $B : (J^rT^*)^* \to (J^rT^*)^*$ over n-manifolds.

Proposition 1. *For integers $n \geq 2$ and $r \geq 2$ any natural transformation $B : (J^rT^*)^* \to (J^rT^*)^*$ over n-manifolds is proportional (by a real number) to the identity natural transformation $id : (J^rT^*)^* \to (J^rT^*)^*$.*

Proof. Clearly, any element from the fibre $(J^rT^*)^*_0\mathbf{R}^n$ is a linear combination of the $(j^r_0(x^\alpha dx^i))^*$ for all $\alpha \in (\mathbf{N} \cup \{0\})^n$ with $|\alpha| \leq r$ and $i = 1, \dots, n$, where the $(j^r_0(x^\alpha dx^i))^*$ form the basis dual to the $j^r_0(x^\alpha dx^i) \in (J^rT^*)_0\mathbf{R}^n$ for α and i as beside.

Any natural transformation B as in the proposition is uniquely determined by the values $\langle B(u), j^r_0(x^\alpha dx^i)\rangle \in \mathbf{R}$ for $u \in (J^rT^*)^*_0\mathbf{R}^n$, $\alpha \in (\mathbf{N} \cup \{0\})^n$ with $|\alpha| \leq r$ and $i = 1, \dots, n$. Since B is invariant with respect to the coordinate permutations, it is uniquely determined by the $\langle B(u), j^r_0(x^\alpha dx^1)\rangle$ for any u and α as above. If $|\alpha| \geq 1$, then the local diffeomorphisms $\varphi_\alpha = (x^1, x^2 + x^\alpha, x^3, \dots, x^n)^{-1}$ sends $j^r_0(x^2dx^1)$ into $j^r_0(x^2dx^1) + j^r_0(x^\alpha dx^1)$. Then (using the invariancy of B with respect to the φ's) B is uniquely determined by the $\langle B(u), j^r_0(x^2dx^1)\rangle \in \mathbf{R}$ and the $\langle B(u), j^r_0(dx^1)\rangle \in \mathbf{R}$ for any $u \in (J^rT^*)^*_0\mathbf{R}^n$.

At first we study the $\langle B(u), j^r_0(dx^1)\rangle \in \mathbf{R}$ for u as above.

By the naturality of B with respect to the homotheties $a_t = (t^1x^1, \dots, t^nx^n)$ for $t = (t^1, \dots, t^n) \in \mathbf{R}^n_+$, $\langle B((J^rT^*)^*(a_t)(u)), j^r_0(dx^1)\rangle = t^1\langle B(u), j^r_0(dx^1)\rangle$ for any $t = (t^1, \dots, t^n) \in \mathbf{R}^n_+$. For any $t \in \mathbf{R}^n$, any $i = 1, \dots, n$ and any $\alpha \in (\mathbf{N} \cup \{0\})^n$ we have $(J^rT^*)^*(a_t)((j^r_0(x^\alpha dx^i))^*) = t^{\alpha+e_i}(j^r_0(x^\alpha dx^i))^*$. Then by the homogeneous function theorem, see [5], we have $\langle B(u), j^r_0(dx^1)\rangle = \lambda u_{(0),1}$ for some $\lambda \in \mathbf{R}$, where $u_{\alpha,i}$ is the coefficient of $u \in (J^rT^*)^*_0\mathbf{R}^n$ corresponding to $(j^r_0(x^\alpha dx^i))^*$,

$(0) = (0,\dots,0) \in (\mathbf{N}\cup\{0\})^n$. Replacing B by $B-\lambda id$, we can assume that $\lambda = 0$, i.e.

$$\langle B(u), j^r_0(dx^1)\rangle = 0 \text{ for any } u \in (J^rT^*)^*_0\mathbf{R}^n\ . \tag{2.1}$$

It remains to show that $\langle B(u), j^r_0(x^2dx^1)\rangle = 0$ for any $u \in (J^rT^*)^*_0\mathbf{R}^n$.

By the naturality of B with respect to the homotheties $a_t = (t^1x^1,\dots,t^nx^n)$ for $t = (t^1,\dots,t^n) \in \mathbf{R}^n_+$ and by the homogeneous function theorem, we have

$$\langle B(u), j^r_0(x^2dx^1)\rangle = \lambda u_{(0),1}u_{(0),2} + \mu u_{e_1,2} + \nu u_{e_2,1} \tag{2.2}$$

for some $\lambda,\mu,\nu \in \mathbf{R}$, $e_i = (0,\dots,0,1,0,\dots,0) \in (\mathbf{N}\cup\{0\})^n$, 1 in i-th position.

It remains to show that $\lambda = \mu = \nu = 0$.

Firstly we prove $\lambda = 0$, i.e. $\langle B((j^r_0(dx^1))^* + (j^r_0(dx^2))^*), j^r_0(x^2dx^1)\rangle = 0$. For, we prove

$$\begin{aligned}
&\langle B((j^r_0(dx^1))^* + (j^r_0(dx^2))^*), j^r_0(x^2dx^1)\rangle\\
&= \langle B((j^r_0(dx^1))^* + (j^r_0(dx^2))^* - \frac{1}{r+1}(j^r_0(x^1)^rdx^1)^*), j^r_0(x^2dx^1)\rangle\\
&= \langle B((j^r_0(dx^2))^* - \frac{1}{r+1}(j^r_0(x^1)^rdx^1)^*), j^r_0(x^2dx^1)\rangle = 0\,.
\end{aligned} \tag{2.3}$$

The first and the third equalities of (2.3) are consequences of formula (2.2). We prove the second equality of (2.3). We observe that the local diffeomorphism $\varphi = (x^1+(x^1)^{r+1}, x^2,\dots,x^n)$ preserves $j^r_0(x^1dx^2)$, $(j^r_0(dx^1))^*$ and $(j^r_0(dx^2)^*$, and it sends $(j^r_0((x^1)^rdx^1)^*$ into $(r+1)(j^r_0(dx^1))^* + (j^r_0((x^1)^rdx^1))^*$. (For example, we prove the last fact. Given $\alpha \in (\mathbf{N}\cup\{0\})^n$ with $|\alpha| \le r$ and $i = 1,\dots,n$ we have $l := \langle (J^rT^*)^*(\varphi)((j^r_0((x^1)^rdx^1))^*), j^r_0(x^\alpha dx^i)\rangle = \langle (j^r_0((x^1)^rdx^1))^*, j^r_0(\varphi_*^{-1}(x^\alpha dx^i))\rangle = \langle (j^r_0((x^1)^rdx^1))^*, j^r_0(x^\alpha dx^i) + (r+1)\delta^i_1 j^r_0(x^\alpha(x^1)^rdx^i)\rangle$. Then l is equal to 1 if $j^r_0(x^\alpha dx^i) = j^r_0((x^1)^rdx^1)$, l is equal to $r+1$ if $j^r_0(x^\alpha dx^i) = j^r_0(dx^1)$, and l is equal to 0 in the other cases.) Now, using the naturality of B with respect to φ we end the proof of the second equality of (2.3).

Now, we show $\mu = 0$, i.e. $\langle B((j^r_0(x^1dx^2))^*), j^r_0(x^2dx^1)\rangle = 0$. For, we prove if $r \ge 2$, then

$$\begin{aligned}
&\langle B((j^r_0(x^1dx^2))^*), j^r_0(x^2dx^1)\rangle\\
&= \langle B((j^r_0(x^1dx^2))^* + 2(j^r_0((x^1)^2dx^2))^*), j^r_0(x^2dx^1)\rangle\\
&= \langle B(2(j^r_0((x^1)^2dx^2))^* + (j^r_0(dx^2))^*), j^r_0(x^2dx^1)\rangle = 0\,.
\end{aligned} \tag{2.4}$$

The first and the third equalities of (2.4) are consequences of formula (2.2). We prove the second equality of (2.4). It is not difficult to verify that the local diffeomorphism $\psi = (x^1+\frac{1}{2}(x^1)^2, \frac{x^2}{1+x^1}, x^3,\dots,x^n)$ preserves $j^r_0(x^2dx^1)$, it sends $(j^r_0(x^1dx^2))^*$ into $(j^r_0(x^1dx^2))^* - (j^r_0(dx^2))^*$, and it sends $(j^r_0((x^1)^2dx^2))^*$ into $(j^r_0((x^1)^2dx^2))^* - \frac{1}{2}(j^r_0(x^1dx^2))^* + (j^r_0(dx^2))^*$. Now, using the naturality of B with respect to ψ we end the proof of the second equality of (2.4).

It remains to show $\nu = 0$, i.e. $\langle B((j^r_0(x^2dx^1))^*), j^r_0(x^2dx^1)\rangle = 0$. We see

$$
\begin{aligned}
0 =&\langle B((j^r_0(x^1dx^2))^*), j^r_0(dx^1)\rangle \\
&= \langle B((j^r_0(x^1dx^2))^* - (j^r_0(dx^1))^*), j^r_0(dx^1) + j^r_0(x^1dx^2) + j^r_0(x^2dx^1)\rangle \\
&= \langle B((j^r_0(x^1dx^2))^* - (j^r_0(dx^1))^*), j^r_0(x^1dx^2)\rangle \\
&= \langle B((j^r_0(x^2dx^1))^* - (j^r_0(dx^2))^*), j^r_0(x^2dx^1)\rangle \\
&= \langle B((j^r_0(x^2dx^1))^*), j^r_0(x^2dx^1)\rangle\,.
\end{aligned} \tag{2.5}
$$

The first equality of (2.5) is a consequence of formula (2.1). The fifth one is a consequence of formula (2.2). The fourth one is a consequence of the naturality of B with respect to the diffeomorphism $(x^2, x^1, x^3, \dots, x^n)$ permuting x^1 and x^2. The third one is a consequence of formulas (2.1), (2.2) and (2.4). We prove the second equality of (2.5). It is not difficult to verify that the local diffeomorphism $\theta = (x^1 + x^1x^2, x^2, \dots, x^n)^{-1}$ sends $j^r_0(dx^1)$ into $j^r_0(dx^1) + j^r_0(x^1dx^2) + j^r_0(x^2dx^1)$ and it sends $(j^r_0(x^1dx^2))^*$ into $(j^r_0(x^1dx^2))^* - (j^r_0(dx^1))^*$. Now, using the naturality of B with respect to θ we end the proof of the second equality of (2.5).

The proof of Proposition 1 is complete. □

3. We are now in position to prove Theorem 1. Clearly, the natural operators $(J^rT^*)^*$, L and $A^{(s)}$ for $s = 1, \dots, r+1$ are linearly independent. So, it is sufficient to show that for integers $n \geq 2$ and $r \geq 2$ any natural operator $A : T_{|\mathcal{M}_n} \rightsquigarrow T(J^rT^*)^*$ is a linear combination with real coefficients of the natural operators $(J^rT^*)^*$, L and $A^{(s)}$ for $s = 1, \dots, r+1$

Let $A : T_{|\mathcal{M}_n} \rightsquigarrow T(J^rT^*)^*$ be a natural operator, where $r \geq 2$ and $n \geq 2$ are integers.

The G^{r+1}_n-space $S = (J^rT^*)^*_0\mathbf{R}^n$ corresponding to $(J^rT^*)^*$ is naturally contracttible to $q = 0 \in S$ in the sense of Def.1 in [4]. Then by Proposition 1 in [4] there exists a number $\lambda_A \in \mathbf{R}$ such that $A - \lambda_A(J^rT^*)^* : T_{|\mathcal{M}_n} \rightsquigarrow T(J^rT^*)^*$ is a vertical operator. Then replacing A by $A - \lambda_A(J^rT^*)^*$ we can assume that

$$A \text{ is a vertical operator.} \tag{3.1}$$

Define a natural transformation $B_A := pr_2 \circ A(0) : (J^rT^*)^*M \to (J^rT^*)^*M$ for any n-manifold M, where 0 is the zero vector field on M and $pr_2 : V(J^rT^*)^*M \tilde{=} (J^rT^*)^*M \times_M (J^rT^*)^*M \to (J^rT^*)^*M$ is the projection onto second factor. By Proposition 1, there exists $\mu_A \in \mathbf{R}$ such that $B_A = \mu_A id$. Then replacing A by $A - \mu_A L$ we can assume that

$$A(0) = 0 \in \mathcal{X}((J^rT^*)^*M) \text{ for any } n\text{-manifold } M\,. \tag{3.2}$$

We define $\tilde{A} : \mathbf{R} \times (J^rT^*)^*_0\mathbf{R}^n \to (J^rT^*)^*_0\mathbf{R}^n$, $\tilde{A}(\lambda, u) = pr_2 \circ A(\lambda\partial_1)(u)$, $\lambda \in \mathbf{R}$, $u \in (J^rT^*)^*_0\mathbf{R}^n$, where pr_2 is as above. It is well-known that A is uniquely determined by $\tilde{A}(1, .) = pr_2 \circ A(\partial_1)_{|(J^rT^*)^*_0\mathbf{R}^n}$. So, we will study $\tilde{A}$.

Let $P(r)$ be the set of all pairs (α, i), where $\alpha \in (\mathbf{N}\cup\{0\})^n$ is such that $|\alpha| \leq r$ and $i = 1,\dots,n$. For any $(\alpha,i) \in P(r)$ we define $\tilde{A}_{\alpha,i} : \mathbf{R} \times (J^rT^*)^*_0\mathbf{R}^n \to \mathbf{R}$ such that $\tilde{A} = \sum_{(\alpha,i)\in P(r)} \tilde{A}_{\alpha,i} \cdot (j^r_0(x^\alpha dx^i))^*$. By the naturality of A with respect to the homotheties $a_t = tid_{\mathbf{R}^n}$ for $t \in \mathbf{R}_+$ we have the homogeneity condition $\tilde{A}_{\alpha,i}(t\lambda, (J^rT^*)^*_0(a_t)(u)) = t^{|\alpha|+1}\tilde{A}_{\alpha,i}(\lambda, u)$ for any $\lambda \in \mathbf{R}$, any $u \in (J^rT^*)^*_0\mathbf{R}^n$ and any $(\alpha, i) \in P(r)$. By (3.2), $\tilde{A}_{\alpha,i}(0,.) = 0$ for any $(\alpha,i) \in P(r)$. Now, by the homogeneous function theorem, $\tilde{A}_{\alpha,i}(\lambda,u)$ is a linear combination of monomials in λ and the $u_{\beta,j}$ for $(\beta,j) \in P(r)$ with $|\beta| \leq |\alpha|-1$, where given u the $u_{\beta,j}$ are the coordinates of u as in Item 2. Hence for all $\mu_{\beta,j} \in \mathbf{R}$ we have

$$\tilde{A}(1, \sum_{(\beta,j)\in P(r)} \mu_{\beta,j} \cdot (j^r_0(x^\beta dx^j))^*) = \tilde{A}(1, \sum_{(\beta,j)\in P(r-1)} \mu_{\beta,j} \cdot (j^r_0(x^\beta dx^j))^*) \ . \tag{3.3}$$

We prove that $\tilde{A}(1,u) = \tilde{A}(1,0)$ for all $u \in (J^rT^*)^*_0\mathbf{R}^n$.

Assume the contrary. Let k be the minimal number such that there exists $(\beta^o, j^o) \in P(r)$ with $|\beta^o| = k$ such that $\Phi((\mu_{\beta,j})_{(\beta,j)\in P(r)}) := \tilde{A}(1, \sum_{(\beta,j)\in P(r)} \mu_{\beta,j} \cdot (j^r_0(x^\beta dx^j))^*)$ depends essentially on μ_{β^o,j^o}, i.e. $\frac{\partial}{\partial \mu_{\beta^o,j^o}}\Phi \neq 0$. (Then $r-k \geq 1$.) We fix some (β^o, j^o) as above such that β^o_n is minimal, where $\beta^o = (\beta^o_1,\dots,\beta^o_n)$.

We produce a contradiction.

We say that $(j^r_0(x^\beta dx^j))^*$, where $(\beta,j) \in P(r)$, is not essential if $|\beta| < k$ or $|\beta| = r$ or ($|\beta| = k$ and $\beta_n < \beta^o_n$). Let $\varphi = (x^1,\dots,x^{j^o} + (x^n)^{r-k+1},\dots,x^n)$ (only the j^o-position is exceptional) be a local diffeomorphism. Denote $\tilde{\varphi} := (J^rT^*)^*_0(\varphi)$ and $\tilde{A}_1 = \tilde{A}(1,.)$. It will be proved below that

$$\begin{aligned}
\Phi((\mu_{\beta,j})_{(\beta,j)\in P(r)}) &= \tilde{A}_1(\sum_{(\beta,j)\in P(r-1),|\beta|\geq k} \mu_{\beta,j} \cdot (j^r_0(x^\beta dx^j))^*) \\
&= \tilde{A}_1(\sum_{(\beta,j)\in P(r-1),|\beta|\geq k} \mu_{\beta,j} \cdot (j^r_0(x^\beta dx^j))^* - \frac{\mu_{\beta^o,j^o}}{a}(j^r_0(x^{\beta^o}(x^n)^{r-k}dx^n))^*) \\
&= \tilde{\varphi}^{-1}\circ\tilde{A}_1(\tilde{\varphi}(\sum_{(\beta,j)\in P(r-1),|\beta|\geq k} \mu_{\beta,j} \cdot (j^r_0(x^\beta dx^j))^* - \frac{\mu_{\beta^o,j^o}}{a}(j^r_0(x^{\beta^o}(x^n)^{r-k}dx^n))^*)) \\
&= \tilde{\varphi}^{-1}\circ\tilde{A}_1(\sum_{(\beta,j)\in P(r-1),|\beta|\geq k} \mu_{\beta,j} \cdot (j^r_0(x^\beta dx^j))^* - \mu_{\beta^o,j^o}\cdot(j^r_0(x^{\beta^o}dx^n))^* + \dots) \\
&= \tilde{\varphi}^{-1}\circ\tilde{A}_1(\sum_{(\beta,j)\in P(r-1),|\beta|\geq k} \mu_{\beta,j} \cdot (j^r_0(x^\beta dx^j))^* - \mu_{\beta^o,j^o}\cdot(j^r_0(x^{\beta^o}dx^n))^*) \ ,
\end{aligned}$$

where the dots is the linear combination of the not essential $(j^r_0(x^\beta dx^j))^*$'s and $a \neq 0$ is some real number which will be defined bellow. Then Φ is independent of μ_{β^o,j^o}, i.e. we have a contradiction.

Let us explain the above equalities.

The first, the second and the last equalities are consequences of the formula (3.3), the definition of k, the definition of (β^o, j^o), the definition of not essential $(j^r_0(x^\beta dx^j))^*$'s and the equality $|\beta^o|+r-k = r$. The third equality is a consequence of the invariancy of A and ∂_1 with respect to φ. The fourth equality is a consequence of the following (not difficult to verify) two facts. For any $(\beta, j) \in P(r-1)$ with $|\beta| \geq k$ the local diffeomorphism φ sends $(j_0^r(x^\beta dx^j))^*$ into $(j_0^r(x^\beta dx^j))^* + \dots$, where the dots denote the linear combination of the $(j_0^r(x^\alpha dx^i))^*$ for $|\alpha| < k$. The local diffeomorphism φ sends $(j_0^r(x^{\beta^o}(x^n)^{r-k} dx^n))^*$ into $a \cdot (j_0^r(x^{\beta^o} dx^{j^o}))^* + \dots$ for some real number $a \neq 0$, where the dots denote the linear combination of $(j_0^r(x^{\beta^o}(x^n)^{r-k} dx^n))^*$, $(j_0^r(x^{\beta^o + e_{j^o} - e_n} dx^n))^*$ (only in the case $j^o \neq n$ and $\beta_n^o \neq 0$) and the $(j_0^r(x^\alpha dx^i))^*$ for $|\alpha| < k$.

We have proved that $\tilde{A}(1, u) = \tilde{A}(1, 0)$ for any $u \in (J^r T^*)_0^* \mathbf{R}^n$. Now, using the invariancy of A and ∂_1 with respect to the $b_t = (x^1, tx^2, \dots, tx^n)$ for $t \in \mathbf{R}_+$ and next putting $t \to 0$ we deduce that $\tilde{A}(1, u) = \sum_{i=0}^r \lambda_i \cdot (j_0^r((x^1)^i dx^1))^*$ for any $u \in (J^r T^*)_0^* \mathbf{R}^n$, where λ_i for $i = 0, \dots, r$ are some real numbers. Hence the vector space of all natural operators $A : T_{|\mathcal{M}_n} \rightsquigarrow T(J^r T^*)^*$ satisfying conditions (3.1) and (3.2) has dimension $\leq r + 1$. On the other hand the operators $A^{(s)}$ for $s = 1, \dots, r+1$ satisfy (3.1) and (3.2), and they are linearly independent. Therefore our A is a linear combination of the $A^{(s)}$ for $s = 1, \dots, r+1$.

The proof of Theorem 1 is complete. □

Acknowledgement. This paper is supported by the grant of KBN.

References

[1] Doupovec, M., *Natural operations transforming vector fields to the second order tangent bundle*, Čas. pěst. mat. 115(1990), 64–72.

[2] Gancarzewicz, J., *Lifting of functions and vector fields to natural bundles*, Dissert. Math. CCXII, Warszawa, 1983.

[3] Kolář, I., *On natural operators on vector fields*, Ann. Global Analysis and Geometry, 6(2) (1988), 109–117.

[4] Kolář, I., *On the natural operators transforming vector fields to the r-th tensor power,*, Suppl. Rendiconti Circolo Mat. Palermo, 32 (II) (1993), 15–20.

[5] Kolář, I., Michor, P. W., Slovák, J., *Natural operations in differential geometry*, Springer-Verlag, Berlin 1993.

[6] Kobak, P., *Natural liftings of vector fields to tangent bundles of bundles of 1-forms*, Math. Boch. 116 (1991), 319–326.

[7] Mikulski, W. M., *Some natural operations on vector fields*, Rendiconti Math. Roma 12(VII) (1992), 783–803.

[8] Mikulski, W. M., *Some natural constructions on vector fields and higher order cotangent bundles*, Mh. Math. 117 (1994), 107–119.

[9] Tomáš, J., *Natural operators on vector fields on the cotangent bundles of the bundles of (k,r)-velocities*, Rend. Circ. Mat. Palermo 5431(II) (1998), 113–124 239–249.

Institute of Mathematics, Jagiellonian University
Kraków, Reymonta 4, POLAND
E-mail: Wlodzimierz.Mikulski@im.uj.edu.pl